

The Overlooked Dimensions of Electoral Systems

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ABSTRACT

The number of seats per district (district magnitude) is a critical component of electoral systems, affecting key outcomes such as legislative fragmentation, representation of minorities, and accountability. We point to an overlooked aspect of districts which serves as input to a silent first step in any cross-country study involving districted electoral systems. We show that any single electoral system employing districts can be characterized by three alternative units of analysis that capture its districting structure: partition to districts themselves, the assignment of representatives to districts, and the assignment of voters to districts. Importantly, we demonstrate that the three often result in different characterizations of the same electoral system, and that these differences increase with within-country variation in magnitude and malapportionment, common in Europe and Latin America, respectively. We theorize about the circumstances under which each unit should be used and identify a common malpractice in the characterization of electoral systems.

Word count: 3943

1. INTRODUCTION

Transforming voters' voices into seats, electoral systems are arguably the most important institutional mechanism shaping politics in representative democracy. While this mechanism is defined by a broad set of pieces, districts are perhaps the key piece defining an electoral system. District magnitude, the number of seats per district, shapes the environment under which electoral competition takes place, thereby affecting a wide variety of political outcomes. Among others, district magnitude affects party system fractionalization (Clark and Golder 2006; Neto and Cox 1997), policy formation process (Lijphart 1999), disproportionality in representation (Gallagher 1991), congruence between voters and representatives (Huber and Powell 1994; Powell 2000; Powell and Vanberg 2000), redistribution (Iversen and Soskice 2006), female parliamentary representation (Matland 1993), and strategic voting (Cox 1997; Duverger 1963). It is therefore well established that district magnitude is a fundamental characteristic that shapes how voters cast their ballots and how candidates compete and representatives deliver.

The vast majority of democratic polities are divided into districts, and thus in cross-country studies involving electoral systems the ubiquitous first step taken is a summary characterization of each country's district magnitudes. The researcher summarizes (or uses some off-the-shelf summary of) the information regarding a country's districts to a single score in order to characterize the country as a whole. Once each country has a score, the political outcome in question is analyzed against these scores. In the small subset of countries where all districts are of the same magnitude – most notably, the Anglo-American model of single-member districts – this summary is straightforward. Under the most prevalent electoral system in the

democratic world, however – proportional representation with districts – variation in magnitude is a commonplace: within the same country, different districts have different numbers of seats, with the gap between small and large sometimes reaching a gap between several and several dozen (e.g., Argentina’s La Rioja district with 3 and Buenos Aires with 35 seats). How should one summarize and characterize the variety of districts in comparative cross-country studies? And consequently, what would the expectation regarding each of the outcomes above and many others be where some districts elect few or even only a single candidate and others elect many?

In this essay, we lift the hood over the excruciatingly important and – in our mind, mistakenly – non-glorious stage of summarizing an electoral system. Summarizing an electoral system, we argue, is first and foremost a theoretical praxis rather than a mechanical nuisance. We theorize about it, show why it matters, call attention to malpractices, and propose an alternative. We show that an electoral system with a given districting structure can be summarized by three different units: districts, elites, and voters. Simply put, summarizing an electoral system by one unit vs. the other can result in substantially different characterizations of it.

Our contribution is twofold. First, we theorize about the units (dimensions) by which an electoral system should be characterized. We further identify common practices in the selection of units which we argue are misleading, and provide analytic guidance on the units by which electoral systems should be characterized in different contexts. Second, we identify the conditions under which summaries of a given electoral system by different units will result in different scores. Specifically, we empirically demonstrate that variation in district magnitude (common in European

democracies) and malapportionment (common in Latin America) lead to substantial discrepancies between characterizations based on different units.

2. THE THREE DIMENSIONS OF DISTRICTING

One might wonder how different conceptualizations of districting in a given country would result in empirically different characterizations of it to begin with. After all, at any given time, there is only one partition per country. A simple illustration may shed light on this issue. Consider a parliament of 11 seats and three districts: D1 and D2 have a single seat each and D3 has nine seats, a distribution of magnitudes that resembles that of many countries (e.g., Norway, Cyprus, Brazil, and Poland).

The seemingly most obvious unit by which to characterize and code this system is districts themselves: two districts elect a single seat each and a third district elects nine seats. Drawing on this unit, one can utilize various statistics: two-thirds of the districts are single-member, the median district is of a single seat, and the average district is of 3.67 seats. Our argument, however, is about the choice of unit of analysis drawn upon, rather than the choice of summary statistic of it. It holds regardless of the latter.

We contend that districts themselves are not only irrelevant for most questions, but are also often misleading. Although two-thirds of the districts in this system are of a single member and governed by majoritarian logic, the vast majority of actors in this system – both elites and voters – operate in a multi-member district environment and therefore by proportional representation. Summarizing this system by districts will lead us to infer that it is mostly majoritarian, while taking into account the same districting structure yet incorporating it through the eyes of the actors

involved reveals that it is mostly proportional. Districts are merely the vessel into which representatives and voters are assigned. To summarize this electoral system by a politically relevant dimension, one ought to focus on the allocation (districting) of actors – elites or voters – to these vessels, not the vessels themselves.

We propose characterizing the system by focusing on the assignment, in fact the districting of either elites or voters, depending on the research question at hand. This is not a proposed shift from an institution to behavior of agents, but rather it calls for the incorporation of the institution into our analyses of behavior.

Districting of representatives. The first unit is representatives. How are representatives assigned to districts in this electoral system? Take the average representative as an example: she is elected in a district of 7.5 seats (two of the eleven representatives are elected in a district of a single seat and nine in a district of nine seats) as opposed to the average district of 3.67 seats.

This unit is relevant for understanding phenomena where the core of the explanation takes place at the elite level. Thus, if we hypothesize that the number of female candidates has to do with within-party competition for viable candidacy spots, and large districts reduce the costs of that competition and allow for a more “balanced” ticket (Matland 1993; Schwindt-Bayer and Mishler 2005), summarizing the electoral system by districting of representatives rather than districting in itself is a promising way forward.

Districting of voters. An alternative unit of analysis by which we can summarize this system is the allocation of voters to districts. In many countries, the ratio of eligible voters to seats is unequal across districts throughout the country: voters residing in districts of a smaller (larger) ratio are overrepresented

(underrepresented). In a malapportioned country there is discrepancy between the allocation of voters and that of representatives to districts. This is particularly common in Africa and Latin America (Kamahara et al. 2021; Samuels and Snyder 2001).

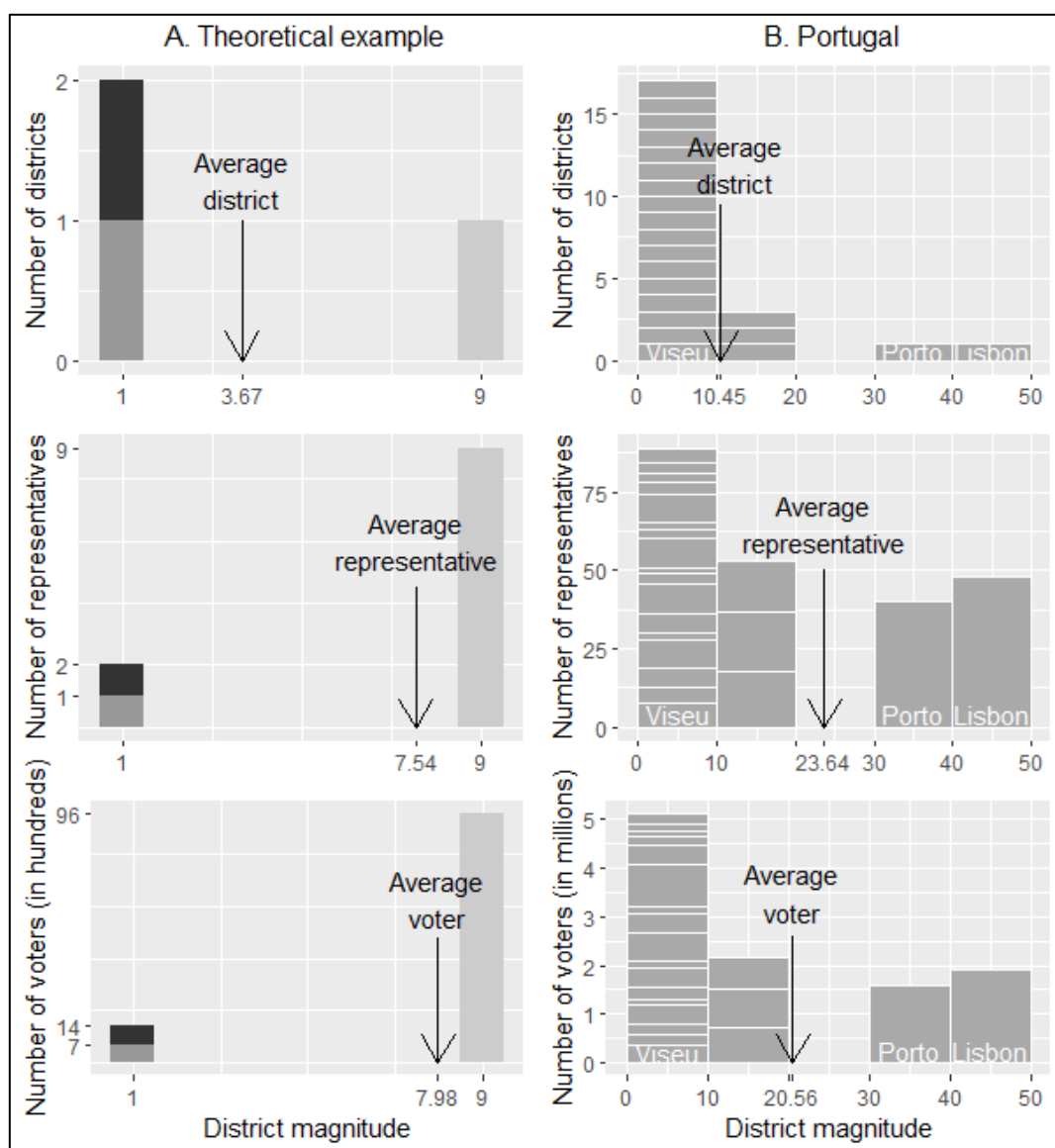
In the presence of malapportionment, the two units will produce different results. Suppose that the country is malapportioned such that 700 eligible voters reside in D1 and D2 each, and 9,600 in D3. In this case, the average voter casts her ballot in a district of 7.98 seats. Thus, in a cross-country study, the district of the average representative would approximate the environment in which the average voter votes in some of the countries, while it would fail to do so in the more malapportioned ones. Following up on our example above, we call for the use of this unit when seeking to understand phenomena where the core of the explanation takes place at the voters' court. Thus, if we seek to understand strategic voting and hypothesize that when casting their ballots voters take into consideration how their vote affects policy outcome – an effect that varies dramatically by district magnitude – the relevant unit by which to summarize the electoral system is districting of voters.

Figure 1a presents the three summaries of this hypothetical electoral system. On the horizontal axis is district magnitude, and on the vertical one is the unit by which the system is summarized. The top panel presents the electoral system by its districts, hence giving each district an equal weight regardless of its magnitude: two districts of a single seat and one of nine seats. The second panel summarizes the electoral system by representatives, showing eleven representatives, two of which are elected in a magnitude of one and nine in a magnitude of nine. This is equivalent to weighing each district by the number of representatives they each elect. It is not surprising, therefore, that the magnitude of the district electing the average representative is

substantially greater than that of the average district. Lastly, the bottom panel summarizes the electoral system by voters, focusing on the malapportioned example above. It shows 1400 voters casting their ballots in single-member districts and 9600 in a nine-member district, and is akin to weighing each district by the number of voters in it. Because of this malapportioned allocation of voters, the average voter casts her ballot in a district larger than that of the average representative.

In summary, in this electoral system, two-thirds of the districts embody the majoritarian model of democracy, but only 0.18 of representatives represent, and 0.13 of voters cast their ballots in them. This simple exercise demonstrates that each of the three units can result in a different summary of the same electoral system, and in particular, when seeking to understand either elite or voter behavior, summarizing an electoral system by districts themselves can lead us astray. The institution of districts should be incorporated into the analysis not in itself but rather as it pertains to the relevant political actors: elites or voters.

FIGURE 1. Summaries of electoral systems



Note: the figure presents the number of districts (top), representatives (middle), and voters (bottom) by their district magnitude. Column A does so for our hypothetical example, and column B for Portugal 2019.

Figure 1b presents an empirical version of the theoretical example above, focusing on Portugal's 22-district, 230-seat parliament. The top panel draws on districts to present the system. The average district is of 10.5 seats. Note that given the equal weight for each district under this characterization, the districts of Porto (40)

and Lisbon (48) combined, just like any other two districts, consist of 0.09 (2/22) of the districts. The second panel draws on representatives as the relevant unit of the Portuguese electoral system. The average representative is elected in a district of 23.6 seats – more than twice as large as the average district. Although 0.09 of the districts, representatives of Porto (40) and Lisbon (48) consist of 0.38 (88/230) of the parliament. Lastly, the bottom panel draws on voters as the relevant unit. The average voter casts her ballot in a district of 20.6 seats. That on average voters reside in smaller districts and representatives are elected in larger ones is consistent with malapportionment whereby small districts (in this case of citizens living abroad) are underrepresented at the expense of large ones.

Before we proceed, let us survey the way electoral systems are summarized in the field of comparative politics. Cross-country studies characterize districted systems in a variety of ways. The most common summary, however, draws on districts as the unit of analysis, almost always with no justification. A review of a decade of publications (2011-2021) in four leading journals in the discipline, both general and comparative (*AJPS*, *BJPS*, *CPS*, and *ES*) finds that among 52 studies that summarize district magnitude at the national level, 42 do so drawing on districts as the unit by which the electoral system is characterized. Only five of the studies explicitly summarize the districting structure by drawing on representatives as the relevant unit.¹

¹ Of the five remaining studies, two draw on districts yet complement their measure with either the variance of magnitudes (Barceló and Muraoka 2018) or the fraction of legislature elected in multi-member districts (Huber 2012), thus indirectly including

Discussion of the choice of unit by which to characterize the system is therefore scarce. An exception we are aware of, which predates our systematic literature review, is Cox (1997). In his book, Cox departs from the common practice that draws on districts and summarizes electoral systems by drawing on representatives. He illustrates the point by presenting a hypothetical example with large variation in magnitude, and explains that summarizing an electoral system by districts might lead us astray (1997, 208-9; see also Neto and Cox 1997).

While under particular circumstances the three characterizations of an electoral system are identical, they usually differ. In the next section we examine the circumstances under which they differ.

3. EMPIRICAL ILLUSTRATION: DISTRICTS, REPRESENTATIVES, AND VOTERS

Two factors affect the degree to which the three characterizations empirically diverge: variation in district magnitude and malapportionment.

In the analysis that follows, we draw on district-level data from elections in all democracies that score eight or higher on Polity V (Marshall et al. 2019) whose population exceeds 250,000, and employ proportional representation with districts (we exclude national district PR and MMP systems), 38 cases altogether (see on-line appendix table). For each country, we take the most recent election for which we are able to secure data, such that almost all elections included took place in the last decade and the earliest in 2006. Geographically, our cases are in Western Europe, _____ representatives in their conceptualization of districting, and three are unclear on the unit of analysis they draw on.

Eastern Europe, Latin America, Asia, and Africa. We complement the district-level data (Kollman et al. 2019) with official records of election results published by National Election Committees or the Ministry of the Interior.

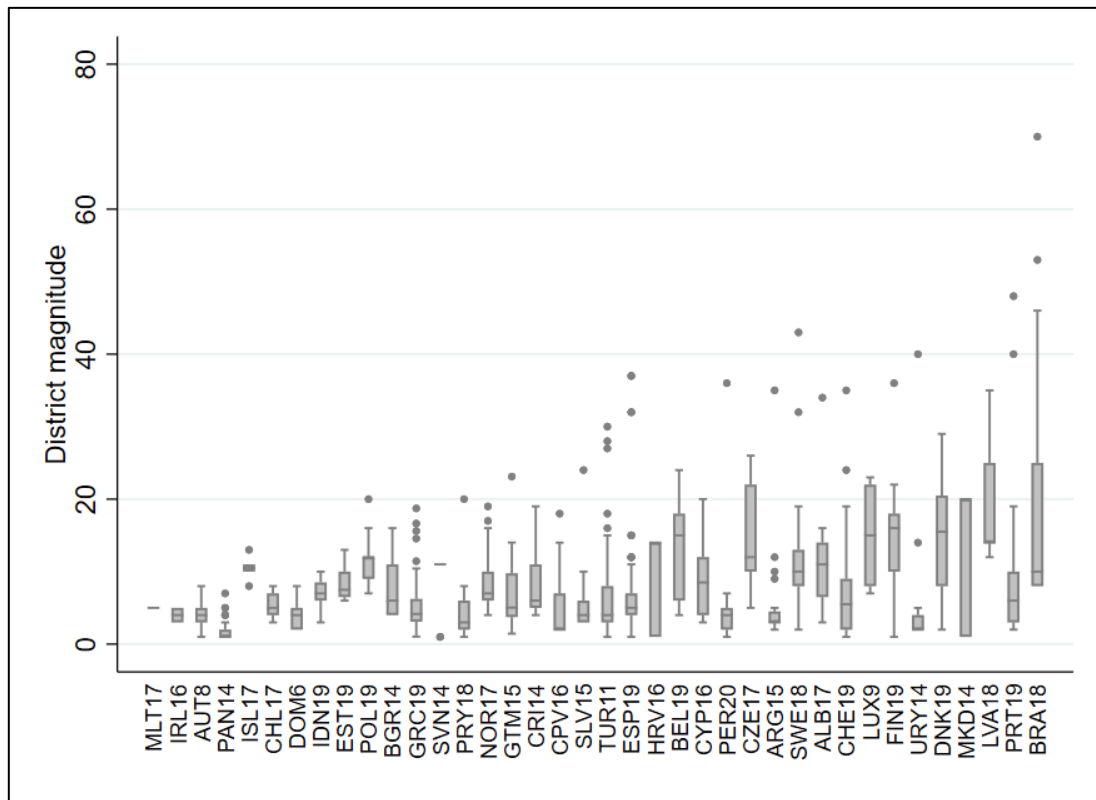
Variation in district magnitude

Under variation in magnitude, the number of seats per district is not constant throughout the country. Under such variation, a commonplace in many of the polities that employ PR with districts, rural areas and small towns are usually represented by many small districts whereas large cities and metropolitan areas are represented by a few large ones.

Figure 2 presents the median and interquartile range (IQR, marked with a box) of district magnitudes in all polities employing PR with districts. It also presents the range of districts between 1.5 IQR above the 75th percentile and 1.5 IQR below the 25th percentile (marked by whiskers) along with the individual outlier districts beyond that range. Countries are organized by ascending order of their standard deviation of district magnitudes, and while in some of the countries there is no or little variation in magnitude across districts, in others there is substantial one.

In almost all cases where there are outlier districts, these are a single or very few large districts, usually urban ones. Turkey (2011) is one such example. While the median province elects four of the 550-seat Grand National Assembly, the three largest provinces (three regions of Istanbul) elect 27, 28, and 30 members each. Portugal (2019) described above is yet another such example. The median magnitude is thus substantially smaller, sometimes by 30 or even 60 seats than the largest ones (e.g., Sweden, Brazil), and the distribution of districts often has a long upper tail.

FIGURE 2. Variation in district magnitudes

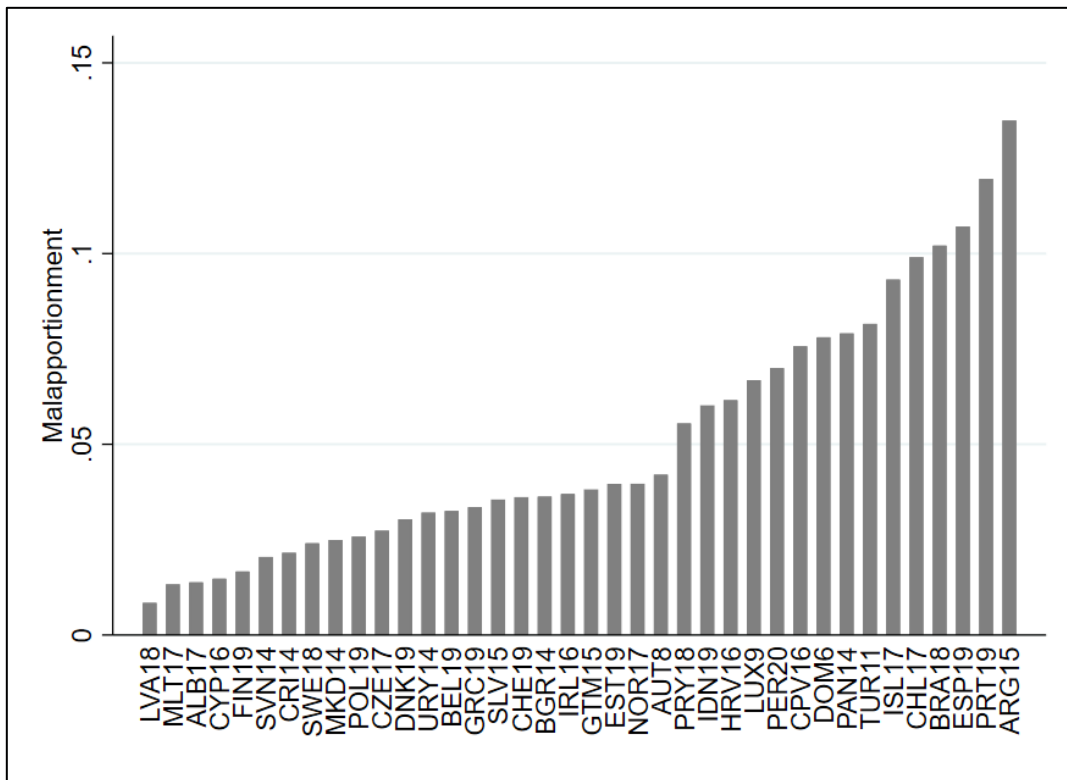


Note: The figure presents the magnitude of the median district (horizontal line), IQR (box), districts within 1.5 IQR above or below the IQR (whiskers), and districts beyond that range (dots), for 38 democracies employing districted PR.

Malapportionment

In the presence of malapportionment, the fraction of voters and that of representatives assigned to a district differ, and thus characterizations of an electoral system by the two units differ. Figure 3 presents the malapportionment in the same set of democracies employing districted PR. The Figure shows substantial malapportionment in general, with fifteen countries having more than five percent of their seats allotted to districts that would not have had these seats otherwise. The figure also shows substantial variation across countries with some (e.g., Malta, Latvia) being close to perfectly apportioned and others (e.g., Brazil, Portugal, Argentina) quite far from it.

FIGURE 3. Malapportionment



Note: Malapportionment in 38 democracies employing districted PR.

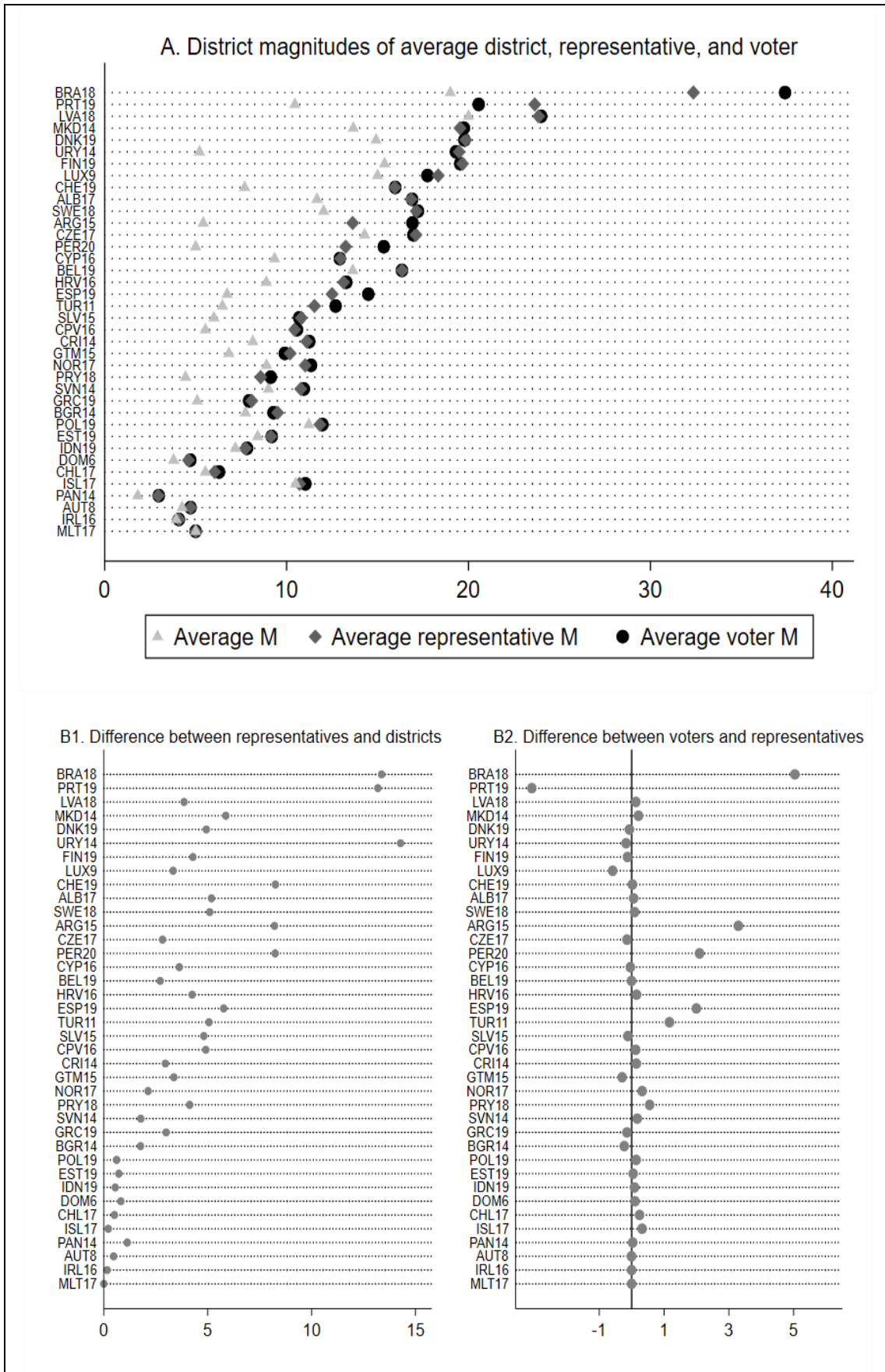
How do characterizations drawing on each of the three units empirically differ from one another? Figure 4a presents the 38 cases reported above, organized (from bottom to top) in ascending order of the standard deviation of their magnitude. On the horizontal axis are the means of the three units for each country: the magnitudes of the average district (marked by triangles), of the district electing the average representative (diamonds), and of the district in which the average voter casts her ballot (circles). The figure demonstrates that the scores of the three dimensions do not exhibit a uniform pattern with mean values of 8.9, 13, and 13.3, respectively.

Panel b1 of Figure 4 presents the differences between the characterization by representatives and that by districts. As the figure shows, in all cases the score of the representative characterization is no smaller than the district one. In several countries

(usually those with no or limited variation) it equals or hovers around zero (e.g., Malta, Ireland, and Iceland), and in many of them it is as large as five or ten seats, and even greater (e.g., Sweden, Switzerland, and Portugal). These are cases with large standard deviation in magnitude and usually a positive skewness with a long upper tail of a few large urban districts and a left hump of small rural districts. Overall, the more varied are district magnitudes, the larger the gap between the magnitude of the district electing the average representative and that of the average district.

Panel b2 presents the differences between the characterization by voters and that by representatives. In many cases the two go hand-in-hand. These are cases in which voters and representatives are similarly assigned to districts and thus malapportionment is limited and the gap between the two hovers around zero. In those cases where the difference is larger (e.g., Spain, Turkey), the average voter casts her ballot in a district greater than that of the average representative. These are cases in which small districts are overrepresented in parliament and get more than the share of representatives they would have gotten by population alone. The opposite is the case in countries where the gap is negative (e.g., Portugal). In these countries the distribution of voters hovers around smaller districts than that of representatives, and so large districts get more representatives than their share of the population. In summary, the type of malapportionment is consistent with the direction of the difference we observe between the two characterizations: where small (large) districts are overrepresented, the characterization of the electoral systems by voters will have a larger (smaller) score than that by representatives.

FIGURE 4. Differences between the three dimensions



4. CONCLUSION: TAKING DISTRICTS SERIOUSLY

Districting is a ubiquitous and highly consequential component of electoral systems. In this note, we showed that what is seemingly an innocuous component is actually an overlooked thorny business, involving both theoretical and empirical aspects. We identified and theorized about the three dimensions of districts and the circumstances under which one should use each, demonstrating that the same districting structure might apply differently to representatives, voters, and districts and lead to three different characterizations of the same electoral system. We showed that within-country variation in magnitude and malapportionment, common in different parts of the world, affect the degree to which the three differ. Last but not least, our analysis points at a common malpractice in the characterization of electoral systems.

While to the best of our knowledge this is the first time the distinction between the three dimensions of districting is systematically made in political science, analogous distinctions are common in other fields. In the field of education, for example, scholars have observed that the average class in a college can be either the class taken by the average student or the class taught by the average faculty, two quantities that almost always differ. The unit of analysis drawn upon, students or faculty members, depends on whether it is the learning or teaching experience that one is interested in (Feld and Grofman 1977).

For the vast majority of substantive questions in comparative politics, districting itself is irrelevant but instead what matters is how it pertains to either representatives, candidates, and parties, or voters, or both. This insight calls to set aside the first dimension, and draw on the other two to characterize electoral systems.

The dimension used to characterize an electoral system ought to be a dimension relevant for answering the research question at hand. Thus, for example, investigating how turnout is affected by competitiveness of a race, one needs to know the number of *voters* casting their ballots under competitive and less competitive contests. Similarly, when investigating how ideological divergence of representatives is shaped by the type of electoral competition they face, one ought to know the fraction of *representatives* elected in small vs. large districts. Approaching any question in comparative politics involving districts, therefore, requires identifying the dimension of districting relevant for the specific question, and answer, at hand. Districts themselves, the vessel into which representatives and voters are distributed, is commonly employed but seldom informative.

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Online Appendix

Descriptive Statistics

Country	Election year	Eligible voters (in thousands)	Total seats	Number of districts	Average district M	Average representative M	Average voter M
Albania	2017	3452	140	12	11.7	16.8	16.9
Argentina	2015	32131	130	24	5.4	13.6	16.9
Austria	2008	6333	183	43	4.3	4.7	4.7
Belgium	2019	8168	150	11	13.6	16.3	16.3
Brazil	2018	146751	513	27	19	32.4	37.4
Bulgaria	2014	6858	240	31	7.7	9.5	9.3
Cape Verde	2016	348	72	13	5.5	10.4	10.6
Chile	2017	14308	155	28	5.5	6	6.3
Costa Rica	2014	3066	57	7	8.1	11.1	11.2
Croatia	2016	3764	151	17	8.9	13.1	13.3
Cyprus	2016	543	56	6	9.3	13	12.9
Czech Republic	2017	8375	200	14	14.3	17.1	17
Denmark	2019	4298	179	12	14.9	19.8	19.8
Dominican Republic	2006	5369	178	47	3.8	4.6	4.7
El Salvador	2015	4912	84	14	6	10.8	10.7
Estonia	2019	887	101	12	8.4	9.1	9.2
Finland	2019	4510	200	13	15.4	19.7	19.5
Greece	2019	9962	300	59	5.1	8.1	7.9
Guatemala	2015	7557	157	23	6.8	10.2	9.9
Iceland	2017	249	63	6	10.5	10.7	11
Indonesia	2019	199708	575	80	7.2	7.7	7.8
Ireland	2016	3304	157	40	3.9	4.1	4.1
Latvia	2018	1549	100	5	20	23.9	24
Luxembourg	2009	224	60	4	15	18.3	17.7
Macedonia	2014	1780	123	9	13.7	19.5	19.7
Malta	2017	342	65	13	5	5	5
Norway	2017	3765	169	19	8.9	11	11.3
Panama	2014	2452	71	39	1.8	2.9	3
Paraguay	2018	4203	80	18	4.4	8.6	9.1
Peru	2020	22902	130	26	5	13.2	15.3
Poland	2019	30254	460	41	11.2	11.8	12
Portugal	2019	10811	230	22	10.5	23.6	20.6
Slovenia	2014	1722	90	10	9	10.8	10.9
Spain	2019	73012	700	104	6.7	12.5	14.5
Sweden	2018	7496	349	29	12	17.1	17.2
Switzerland	2019	5458	200	26	7.7	16	16
Turkey	2011	50216	550	85	6.5	11.5	12.7
Uruguay	2014	2621	99	19	5.2	19.5	19.3