Permissibility of Electoral Systems: A New Look at an Old Question

Orit Kedar*
orit.kedar@mail.huji.ac.il

Liran Harsgor*
liran.harsgor@utoronto.ca

Or Tuttnauer*
or.tuttnauer@mail.huji.ac.il

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Abstract

Permissibility of electoral systems and in particular the conversion of plethora of voices in the electorate to the legislature is broadly considered to depend on the number of seats per district (district magnitude) in a country. Yet the most prevalent electoral system in the democratic world, proportional representation with districts, is often characterized by an almost entirely overlooked variation: within the same country districts vary in their magnitude, sometimes by a factor of twenty. How does such variation affect permissibility of electoral systems? Drawing on a broad cross-section of democracies, we demonstrate that contrary to what the literature implicitly assumes, other things equal, a combination of large and small districts results in greater permissibility than a set of districts of similar magnitude. We find that where districts are of similar (different) magnitude the degree of permissibility is lower (higher) than that found by current literature.

* Orit Kedar is an Associate Professor in the department of political science at the Hebrew University of Jerusalem. Liran Harsgor is a Post-Doctorate Fellow in the Munk School of Global Affairs at the University of Toronto. Or Tuttnauer is a postdoctoral fellow in the department of political science at the Hebrew University of Jerusalem and a visiting researcher in the Bamberg Graduate School of Social Sciences at the University of Bamberg.
Kedar gratefully acknowledges financial support by the ERC. Address: Department of Political Science, The Hebrew University of Jerusalem, Mt. Scopus, Jerusalem, 91905, Israel.

1. Introduction

Given the voice of the electorate, the electoral system – the mechanism aggregating voters' voices and converting them into parties' seats in the legislature – determines the lay of the land in parliament. The permissibility of the electoral system and in particular how the plethora of voices in the electorate are translated to voices in parliament is a key aspect of representation and the focus of this study. We analyze this translation in countries that employ districted systems, focusing on districted proportional representation, the most prevalent electoral system in the democratic world.

It is well established that district magnitude – the number of seats per district – is a key factor determining representation in general and the permissibility of electoral systems in particular. Parliaments elected by proportional representation with districts, however, often (indeed almost always) have varying number of seats across districts. Within the same state, some voters cast their ballots in districts of a few representatives while others in districts of a few dozens. The gap between small and large districts can be zero (e.g. Macedonia), as small as one or two seats (e.g., Iceland) or as large as thirty, forty, or fifty seats (e.g., Sweden, Spain, or Brazil). The voluminous literature on electoral systems notwithstanding, the effect of variation in district magnitude on permissibility – the conversion of plethora of voices in the electorate to legislature – has been unexplored and the variation in itself has been almost entirely ignored.[1]

[1] We are aware of only two studies that examine the fact that different districts are of different magnitudes: Monroe and Rose (2002) and Kedar et al (2016). We discuss these studies below. Neither, however, addresses permissibility as a dependent variable, nor do they examine the same aspects of variation on the right-hand side as those at play in the case of permissibility.

The question motivating this study is: how does within-country variation in district magnitude affect permissibility of electoral systems? We theorize and empirically analyze the effect of variation in magnitude on permissibility and find that in tension with the implicit assumption made by previous studies that it is irrelevant for representation, greater variation in district magnitude leads to greater permissibility. The mechanism is straight forward. The range of districts in a country with substantial variation is often characterized by a long right tail: many districts have few representatives while few have many representatives. Large districts are, by definition, not only more proportionate but also consist of a substantial portion of the legislature. Therefore, the presence of large districts (next to small ones) results in greater permissibility compared with cases in which districts are of similar magnitude.

Drawing on national- and district-level data from a cross section of thirty-eight democracies that employ districted PR and vary in their districting structure as well as additional districted electoral systems, we further demonstrate that ignoring within-country variation in magnitude, previous studies may lead us astray. Where districts are of similar magnitude the translation of plethora of voices in the electorate to the legislature is more distorting, indeed less representative, than previously found. In fact, in countries in which districts are of similar magnitude, larger districts than what one might expect are necessary in order to achieve comparable levels of permissibility.

The paper proceeds as follows. The next section briefly reviews accounts of electoral system conversion of plethora of voices in the electorate to the legislature. The following section presents our argument about the effect of variation in district magnitude on permissibility. The next section presents our empirical analysis. The following section applies

our analysis to another study. The final section concludes and offers implications.

2. Voices in the electorate and their translation to parliamentary seats

Party systems in general and the number of parties in particular have been targets of extensive investigation by students of comparative politics. Although studies of party-system fragmentation in the legislature are driven by various questions and it is not possible to do them all justice here, one can identify two broad themes that receive particular attention. To the best of our knowledge, almost all analyses of the number of parties in legislature include two explanatory factors: (i) the effective number of parties in the electorate (or an account of social heterogeneity that affects it) and (ii) Institutional mediation of the vote – a set of rules of which a specific case is Duverger's mechanical effect.[2] Let us briefly review accounts of institutional mediation in the literature -- the focus of this study.

How do political institutions transform fragmentation in the electorate to that in legislature? Almost all studies analyzing party system in legislature include district magnitude on the right-hand side, and the vast majority of these include the magnitude of a central district. In particular, the literature utilizes the magnitude of the median district (e.g., Carey and Hix 2011), the average district (e.g., Tavits 2008, Shugart et al. 2005), or the district electing the median legislator (e.g., Amorim Neto and Cox 1997) and consistently finds that the greater the central district, the greater the permissibility of the electoral system.

[2]An exception is Taagepera's seat product (Taagepera 2007, see also Li and Shugart 2016) which includes institutional factors alone.

While these three statistics for central tendency measures are the most common account of institutional mediation of the vote, depending on the focus of the study, scholars offer additional ones that either measure central tendency or offer a summary characteristic of the electoral system. Among these measures are the (usually averaged) effective magnitude (Taagepera and Shugart 1993, Taagepera 1999), legal threshold (e.g., Carey and Hix 2011), effective threshold (e.g., Jones 1997, Peñas 2004), proportion of seats in legislature elected via upper tier (e.g., Amorim Neto and Cox 1997, Benoit 2002, Clark and Golder 2006, Stoll 2008), a set of dummy variables categorizing the electoral system to types (Carey and Hix 2011, Kostadinova 2002, Nishikawa and Herron 2004), and compulsory voting (Jensen and Spoon 2011). And while variation in magnitude across districts is ubiquitously present in within-country studies, it is overlooked in cross-country analyses.[3]

3. Institutional mediation of multiple voices

The number of seats allotted to a district, arguably the most important component defining an electoral system, is ubiquitous shorthand for institutional mediation not only in the study of party systems but also in comparative politics more broadly. Yet in most districted PR systems the number of seats per district varies substantially within the same

[3] Two exceptions are Monroe and Rose (2002) and Kedar et al. (2016). The focus of these works, however, is different from the topic of the current study. Monroe and Rose incorporate the variance of the distribution of district magnitudes. In their innovative study, the authors contend that 'first-moment effects concern *how many* players are at the political table. The variance effect determines *which* players are at the table and what cards they hold' (p. 68, emphasis in the original). Kedar et al. analyze the effect of the distribution of magnitudes on inequality in representation.

country. Districts in Sweden vary in magnitude between two and forty-two, in Spain the range is one to thirty-five, and in Brazil the smallest district elects eight seats while the largest one elects seventy. In fact, with three exceptions, all democratic states employing districted PR (thirty-eight in total whose Polity IV score is at least eight) have varying district magnitude.[4]

How varied are district magnitudes within countries, and what shape does such variation take? Figure 1 presents the distribution of districts within districted PR countries. For each country the figure presents all districts in the lower chamber from smallest to largest, such that each rectangle is proportionate to the number of seats in the district and the accumulation of all rectangles is the size of the chamber. Thus, where all districts are of identical magnitude all rectangles are of identical size (e.g., Malta 2008 with sixty-nine seats in total) and where they differ the rectangles on the left are smaller than those on the right. For visual purposes only, cases are divided to groups by parliament size where the first panel includes lower chambers smaller than 110 seats, and the second includes those of 200 seats or greater. To avoid an overly crowded picture the middle group, that of parliaments of size 110-199 seats, is omitted from this figure (but not from those below). It exhibits, however, the same pattern as the other two.

The figure demonstrates several aspects of the variation in district magnitude. First, quite often one or several large districts are present (see the wide rectangles at the right end). Second, when present, they substantially differ in magnitude from most districts. And third, these large districts make up a substantial portion of parliament. Take Brazil 2010 with 513 seats in total as an example. As is evident by the large rectangles on the right, the two

[4]The exceptions are Macedonia, Malta, and Chile.

largest districts (fifty-three and seventy seats) combined consist of 0.24 of its 513-seat legislature, while half of its districts are smaller than ten seats. Similarly, Portugal 2009 has the two largest districts (thirty-nine and forty-seven seats) that sum up to 0.37 of the 230-seat legislature while half of its representatives are elected in districts smaller than six.

Next to each case we note its score on a simple measure that captures the number of legislators elected in large districts compared to the number of those elected in small districts – seat ratio (below we extend our analysis to a variety of measures). Seat ratio is the proportion of seats in parliament elected via districts larger than the median district over the proportion elected in districts smaller than the median district.[5] Thus, in extreme cases where all districts are of identical magnitude (e.g., Malta), the ratio reaches its minimum value of one. The more uneven is the distribution of district magnitude such that some districts are small and others large and thus more legislators are elected via districts larger than the median, the bigger the ratio. Thus, a seat ratio of 3.6 in Brazil 2010 imply that for every representative elected in small districts (smaller than ten, the median), over three are elected in large ones. Similarly, a seat ratio of 4.9 in Portugal implies that the distribution of district magnitudes is so uneven that for every representative elected in a district smaller than six (the median district) almost five are elected in large ones.

Overall, in twenty-nine cases at least sixty percent of legislative seats are elected via half of the districts (a seat ratio score of 1.5 or greater). Sweden 2006 with a median district of eleven seats but also a district as large as forty-two seats is one such example. In nineteen

[5]In cases with an uneven number of districts, we equally divide seats of the median district between the two sections.

countries at least seventy percent of the seats in legislature are elected via half of the districts (a score of 2.33 or greater, e.g., Spain 2008 with a median district of five seats and range of one to thirty-five seats). And in three countries at least eighty percent of the seats in legislature are elected via half of electoral districts (e.g., Switzerland 2007).

-- Figure 1 here --

Needless to say, numerous measures of variation offered by the statistical literature are at our disposal. In the robustness section below we reexamine our analysis that follows employing alternative measures and demonstrate the consistency of our results. The main advantage of the seat ratio, however, is both in its simplicity and in its explicit targeting of large districts. Nonetheless, it strongly correlates with other measures of dispersion (e.g., $r=0.680$, $p<.001$ with standard deviation).

How does the well-familiar median district vary with seat ratio? Figure 2 presents a scatterplot of the two. On the horizontal axis is the magnitude of the median district in each country: half of the districts are smaller and half are larger than the horizontal score, and on the vertical axis is the seat ratio. Each data point in the figure represents a country-year observation. The figure includes all districted PR systems that score eight or higher on Polity IV (as mentioned above, a total of thirty-eight cases). As the figure demonstrates, countries differ in both dimensions. Let us examine the median magnitude first. The median district magnitude varies between one (Panama) and 21 (Italy) and is 8.18 on average. Our cases vary in their average district magnitude (not reported here) as well. The average magnitude varies between 1.2 (Panama) and 22.5 (Italy) and equals 9.3 on average. The degree of

variation in magnitude varies substantially as well, as hinted in Figure 1. Overall, the ratio of legislators elected in large vs. small districts varies between one (these are Chile, Malta, and Macedonia with no variation) and 5.3 (Switzerland), and is 2.5 on average. In over twenty-three of the thirty-eight cases for every legislator elected in districts smaller than the median at least two are elected in larger districts, and in twelve the ratio is at least one to three.

As is visible in the scatterplot, the two dimensions do not cluster (r=-0.2, p-value =0.25). In fact, in many cases countries with similar medians have dramatically different distributions. Next to the scatterplot we hone in on four such examples. Examine Malta and Portugal, with median districts of five and six seats (marked by a black vertical line), respectively, first. In the former all districts are of five representatives each, while Portugal has districts that vary substantially with 19.6 percent of its parliament elected in districts of six representatives or fewer and a long right tail. Iceland and Brazil (with medians of 10.5 and 10, respectively) are another stark example. In the former districts vary between nine and twelve, while the latter is characterized by a long right tail while 22.8 percent of representatives are elected in districts of ten seats or fewer.

-- Figure 2 here --

Although the presence of several large districts might be particularly pronounced in these two examples, it is a general one. In seventy-five percent of the cases presented the average is greater than the median and a similar fraction is characterized by positive skewness, indicating the presence of an upper tail. And while the median, as we report above, is 8.18 on average, about sixty percent of the cases have at least one district of

twenty seats or greater.

The vast literature on electoral politics notwithstanding, the link between within-country variation in magnitude and permissibility has been entirely unexplored. This is the first study that examines the effect of variation and in particular the presence of large districts on the transmission of party system in the electorate to the legislature. Only two studies have paid attention to the effect of variation on representational outcomes, although the focus of both is different from the current one on both the left and the right-hand side. Monroe and Rose (2002) show that the variance effect (measured as the ratio between the standard deviation and the median magnitude) results in advantage for rural interests compared with urban ones and disadvantage for the largest urban party. They also show that parliamentary representation of rural parties in particular is less fragmented due to the variance effect. Kedar et al. (2016) examine inequality in representation and specifically conservative bias in parliament. Their study develops a measure of representational inequality and shows that the fraction of small districts – those employing a quasi-majoritarian electoral system – is a key factor leading to representational inequality. Thus, each of these studies, as well as the one presented in this paper, focus on a different outcome and harness a different aspect of the overlooked within-country variation to explain that outcome.

As the analysis above establishes, within-country variation in magnitude is characterized by a long right tail of several large districts next to a bundle of small districts. This, we argue, has direct implications for the mechanical effect of the electoral system. Since large districts are more proportional than small ones and by definition make up a larger portion of parliament, in the presence of large districts more legislators are elected via more proportional rules than where districts are of similar magnitude. Contrary to what previous

research has implicitly assumed, we contend that variation in magnitude plays a key role in institutional mediation of the vote: an upper tail of large districts likely fosters accurate legislative mirroring of plethora of voices in the electorate. Our hypothesis is thus:

H1. Irrespective of a central district, the bigger the variation in magnitude the more permissible is the electoral system.

This study, therefore, is the first to revisit the mechanical effect on permissibility to include the variation in district magnitude and particularly the upper tail of large districts. We now turn to empirically examine our claim.

4. Empirical analysis: how districts mediate variety of voices in the electorate

4.1. Data

Cases and data. We utilize data on election returns and institutional structure from fifty-four democracies. Since we focus on the effect of districts and particularly that of within-country variation in district magnitude, we run our analysis on districted PR systems (N=38) as well as on a broader set of countries that employ districted electoral systems. The latter includes FPTP (N=9) and mixed-member majoritarian (N=7), altogether N=54. All elections took place between 2005-2012.[6] Cases in which party seats in parliament are assigned via a single national district (both national district PR and mixed-member proportional), however, are not included in the analysis. Geographically, eighteen of our cases are Western European

[6]A single exception is Costa Rica for which we take the 2002 elections and New Zealand for which we take the 1993 elections under FPTP system. A list of countries and election years can be found in Table A1 in the online Appendix.

democracies, ten are Eastern European, twelve are located in Latin America, and the rest in North America, Africa, Asia, the Middle East and Oceania. Our data sources on votes and seats are official records of election results usually published by National Elections Committees or the Ministry of the Interior.

Measurement. In our calculation of the effective number of parties in the electorate (ENPV) and that in legislature (ENPS) we included all competing parties and candidates. For ENPV, we employed Taagepera's method of bounds (1997) for independents and others[7]. Lastly, given the electoral system in Switzerland and Luxembourg in which the number of votes per voter varies across districts, we standardized the total votes per district by the relevant magnitude.

Before shifting to our empirical analysis, a word of caution is in order. For simplicity, we conduct our analyses below with the intuitive and simple measure of seat ratio introduced above. In the penultimate section we conduct the analysis employing four alternative measures of variation plus nine versions of one of the four and find that our results hold. More generally, in this study, we call attention to the overlooked effect of variation and particularly the upper tail of districts on permissibility of electoral systems and thus on representation. We do not argue that one particular measure of within-country variation in district magnitude is an answer for all research questions in electoral politics, nor even for permissibility. Rather, we believe that once variation is taken seriously as a property of electoral systems that potentially affects electoral outcomes, various measures capturing different aspects of it that pertain to different outcomes will be considered.

[7]See 'best practice recipe' in Taagepera (1997), p. 151.

4.2 The mediating effect of small and large districts

How does the districting scheme in a country mediate the conversion of plethora of voices in the electorate to that in legislature? In line with previous work, we allow for key institutional components which have a potential limiting effect on the plethora of voices to interact with social heterogeneity as manifested in electoral fragmentation (e.g., Ordeshook and Shvetsova 1994, Amorim Neto and Cox 1997, Clark and Golder 2006). Also in line with previous work, we include a series of variables to control for whether the electoral system is mixed member majoritarian, fused vote, FPTP system, and its proportion of upper-tier seats.[8] Our principal baseline model is thus:

(1) $ENPS_i = \alpha_0 + \alpha_1 ENPV_i + \alpha_2 ln(MedD_i) + \alpha_3 ENPV_i ln(MedD_i) + \sum_{j=4}^{K} \alpha_j z_{ji} + \upsilon_i$

where $ENPS_i$ is the effective number of parties in legislature in country i, $ENPV_i$ is the effective number of parties in the electorate, $MedD_i$ is magnitude of the median district in country i, z_j is a control variable as specified above, and υ_i is a random error. Results of this model are reported in Model 1 of Table A2 in the on-line appendix. (Table A3 presents an identical set of models run on the thirty-eight districted PR's only). Our initial quantity of interest is extracted from the raw results. Substantively, it is the permissibility of the electoral system: the degree to which plethora of voices in the electorate make their way into parliament. Statistically, it is the marginal effect of ENPV on ENPS, mediated by the magnitude of the median district. This is expressed by:

[8] This specification of controls is quite similar, albeit not identical, to the one employed by Carey and Hix (2011). Among others, Carey and Hix also include a dichotomous variable which they term modified PR: PR with median district magnitude smaller than nine.

$$1(a): \frac{\partial ENPS}{\partial ENPV} = \alpha_1 + \alpha_3 ln(MedD_i).$$

Figure 3(a) presents permissibility on the vertical axis against the magnitude of the median district on the horizontal axis as extracted from Model 1. At the bottom of the figure is a rug plot of districted PR's and MMM systems included in our data. Their logged medians vary between zero in Panama and most MMM systems (median of one) and 3.045 in Italy and South Africa (median of 21).

The figure reveals several things. First, as expected, the effect is positive; *ENPS* is strongly, positively, and significantly correlated with *ENPV*. Other things equal, more parties in the electorate imply more parties in legislature. Second, the relationship increases with median district magnitude and approaches one from below as the median district gets larger. Thus, when district magnitude is small the electoral system has a limiting effect, but the effect dissipates with the increase in district magnitude making the system more permissible and thus the party-system in parliament more similar to that in the electorate. Importantly, both coefficients of the median district (the constitutive and the interaction term) are statistically significant.

Based on our contention above we now shift away from the baseline model to estimating the conversion of plethora of voices to parties in parliament as mediated via both central tendency and variation in district magnitude. The principal model we test is thus:

$$(2)\ ENPS_i = \beta_0 + \beta_1 ENPV_i + \beta_2 ln(MedD_i) + \beta_3 ENPV_i ln(MedD_i) + \beta_4 SR_i + \beta_5 ENPV_i SR_i + \sum_{j=6}^{K} \beta_j z_{ji} + u_i$$

where SR_i is the seat ratio (the ratio of seats elected in the top half of districts over those elected in the bottom half) and u_i is the random error. Results of this estimation are reported

in Model 2 of Table A2. Importantly, in both models the two coefficients of seat ratio (the constitutive term and the interaction) are statistically significant.

Our quantity of interest, the conversion of multiple voices in the electorate to the legislature modified by institutional effects, is presented in Figure 3(b) (extracted from Model 2). On the vertical axis is thus:

(2a) $\frac{\partial ENPS}{\partial ENPV} = \beta_1 + \beta_3 ln(MedD_i) + \beta_5 SR_i$[9]

and on the horizontal axis is the magnitude of the median district. For parsimony, at this stage we set the variation in magnitude to two values. The dotted line stands for countries in which district magnitudes vary substantially (the 90th percentile of seat ratio in our data), and the dashed line stands for cases with little within-country variation (the 10th percentile of seat ratio). Generally, as in the baseline model, the figure presents a positive relationship that increases with median magnitude. The two panels differ, however, in their prediction of how well votes translate into seats given the median district.

Take point A on the left panel, the baseline model. This point represents permissibility level of 0.75. At this level, effectively four parties in the electorate translate into three in the legislature. This permissibility is achieved when the median district is of 6.3 seats (logged median is 1.8). At point A' on the right panel (high seat ratio) this same level of permissibility is achieved via smaller median district (2.5 or logged median of 0.91). Let us compare it to

[9] The estimated standard errors of the two specified effects are thus:

$[var(\hat{\alpha}_1) + (ln(MedD_i))^2 * var(\hat{\alpha}_3) + 2\, ln(MedD_i) * cov(\hat{\alpha}_1 \hat{\alpha}_3)]^{1/2}$ for the baseline model, and

$\left[var(\hat{\beta}_1) + (ln(MedD_i))^2 * var(\hat{\beta}_3) + SR_i^2 * var(\hat{\beta}_5) + 2\, ln(MedD_i) * cov(\hat{\beta}_1 \hat{\beta}_3) + 2SR_i * cov(\hat{\beta}_1 \hat{\beta}_5) + 2\, ln(MedD_i) * SR_i * cov(\hat{\beta}_3 \hat{\beta}_5)\right]^{1/2}$ for our main model.

two cases with little variation (low seat ratio, the dashed line). Point B' represents the same median magnitude. The permissibility achieved at B', however, is 0.53. At this level, effectively four parties in the electorate translate to only two in parliament. C' represents the same level of permissibility as point A. Notice that this level of permissibility is achieved via a much larger median district: 27 seats (logged median of 3.3 seats).

Our initial analysis suggests, then, that variation in magnitude increases permissibility. When the median district is of few seats but a long upper tail of large districts is present (A'), the same level of permissibility is achieved as when the median district is substantially larger but no variation is present (C'). Given the same median district and in the absence of several large districts, however (B'), permissibility is reduced.

-- Figure 3 here --

Figure 4 presents a comprehensive picture of the relationship. The figure presents the median magnitude and seat ratio on the horizontal and vertical axes, respectively. Scattered on this plain are the thirty-eight districted PR's, as in Figure 2. The shades in the body of the figure represent the predicted level of permissibility, the estimated conversion of ENPV to ENPS, collapsed into 0.1 intervals. The darker the shade, the more representative is the electoral system (the predicted value of permissibility approaches one). On the left panel is the value estimated based on Equation 1 (reported in Table A2, Model 1) and calculated in Equation 1a, and on the right panel it is the value estimated based on Equation 2 (reported in Table A2, Model 2) and calculated by Equation 2a. Thus, for every case the figure presents the predicted permissibility of the electoral system by both the baseline and our model.

Let us compare the two panels. As can be seen on the left panel and consistent with results reported above, the party system in legislature more accurately reflects that in the electorate as the magnitude of the median district increases. This is denoted by darker shades as we move to the right. As in current literature, this model is *by design* insensitive to variation in magnitude. Thus, Malta and Portugal have almost identical predicted permissibility (0.72 and 0.74, respectively), as do Iceland and Brazil (0.81 in both).

The right panel presents several interesting findings. First, and controlling for the median district, permissibility increases with seat ratio as the darker shades on the upper part of the figure indicate. Recall that a greater seat ratio between the upper and lower halves of districts implies several large districts next to many small ones. It makes sense, therefore, that greater seat ratio results in greater permissibility and thus greater representativeness. Note in particular the permissibility in countries that differ in their seat ratio but not in their median district. To do so, let us return to the comparisons above. Once variation is incorporated, plethora of voices in the electorate translate substantially better to parliament in Portugal than in Malta (0.93 and 0.59, respectively), and in Brazil than in Iceland (0.87 and 0.68 respectively). Our hypothesis, therefore, finds support in the data.

Second, once variation is included in the model, permissibility only moderately increases with median magnitude (and the statistical significance of the effect is somewhat reduced). This is indicated by the rather slow change in shades that accompanies a move to the right compared to the change on the left panel. Third, notice that given that both the magnitude of median district and variation are at work per the diagonal pattern of shades, de facto, a combination of low median and large variation (top left region of the figure) is exchangeable with that of high median and low variation (bottom right region).

Permissibility in Iceland (seat ratio of 1.17 with a median of 10.5) is similar to that in El Salvador (seat ratio of 2.05 with a median of three seats).

Lastly, and related to previous points, notice that for countries with low seat ratio – those on the bottom part of the figure – the right panel predicts lower permissibility compared with the left one. In other words, this finding suggests that in countries with low variation in magnitude, parliamentary representation achieved is lower than current models assume – standard models *overestimate* how well the party system in the electorate is converted to that in legislature. For countries with high variation (those on the upper part of the figure) the right panel predicts better conversion of votes to seats. In other words, when districts vary in their magnitude, the standard model *underestimates* the quality of conversion of votes to seats.[10]

-- Figure 4 here --

4.3. Drawing an inference: the overlooked effect of large districts

We proceed to analyze the degree to which the electoral system allows multiple voices in the electorate to penetrate parliament (per Equations 1a and 2a) and calculate the difference in predicted permissibility between the standard model and our main model. This is calculated by the gaps between the two estimated effects – 1(a) minus 2(a), with each applied to the particular values of the electoral system (median district and seat ratio) in each country:

[10]Following Braumoeller (2004), footnote 6, we also considered a triple interaction model. The results of this model are provided in Table A4 and Figures A2 and A3 in the appendix.

(3) $\hat{\alpha}_1 + \hat{\alpha}_3 ln(MedD_i) - (\hat{\beta}_1 + \hat{\beta}_3 ln(MedD_i) + \hat{\beta}_5 SR_i)$.

A positive difference implies that the standard model overestimates the conversion compared with our model – permissibility is lower than currently assumed -- and a negative one implies that the standard model underestimates it.

Figure 5 presents this difference with the gap (per Equation 3) on the vertical axis and seat ratio on the horizontal axis. Take Iceland and Portugal as an example. In Figure 4 the standard model predicts a conversion of 0.81 in Iceland (this is the predicted value based on Equation 1a) while our model predicts a conversion of 0.68 (the analogous quantity based on Equation 2a). It thus scores +0.13 – the standard model *overestimates* its permissibility. Similarly, the standard model predicts a conversion of 0.74 for Portugal and our model a conversion of 0.93. It thus scores -0.19 – the standard model *underestimates* its permissibility.

The figure shows that in most cases the standard model overestimates the degree to which votes are converted to seats, and the electoral system is thus less representative than inferred. This is contingent, however, on variation in magnitude. For cases in which districts are of similar magnitude (e.g., Iceland), the standard model infers that more voices in the electorate make their way into parliament than our model indicates --- the electoral system is more limiting than one might infer based on current models (positive difference). For cases with large variation and particularly several large districts (e.g., Portugal), the standard model errs on the conservative side, and the electoral system is, in fact, more permissible than one would infer by the magnitude of a central district alone (negative difference). This is because when the distribution of district magnitudes is stretched upward and substantial part of the legislature is elected via large district(s), the relative weight of small districts in

shaping the legislature decreases and hence the electoral system is more penetrable to multiple parties than the central district suggests. Lastly, note that the differences are all statistically different from zero (ninety-five percent confidence intervals are marked). As expected, the confidence intervals are wider on both extremes of the distribution. This is because the standard error of the difference is quadratic in seat ratio.[11]

Overall, then, the standard model wrongly infers what level of permissibility is accomplished by different median districts. In countries with little variation permissibility is in fact lower than common wisdom suggests, and in those with large variation it is higher.

-- Figure 5 here --

5. Robustness: alternative measures, alternative specifications

The analysis presented so far relies on a single (simple) measure of variation in district magnitude and in particular the upper tail: the share of legislature elected in the top half vs. that elected in the bottom half of districts. In this section, we turn to reconduct our analysis employing alternative measures of variation, alternative model specifications, and a subset of the cases. In a nutshell – our results hold in almost all configurations.

For alternative measures of variation, we utilized (a) the proportion of seats elected in the upper half of districts as a fraction of the total number of seats in legislature, (b) the standard deviation of district magnitude, (c) skewness, and (d) the proportion of legislature

[11] See, e.g., Equation 8 in Brambor et al. 2006. In Footnote 9 we specify the standard errors of both 1a and 1b. We simulate the standard error of the difference 1a-1b via bootstrap over 5,000 draws.

elected in large districts, defined in absolute terms. We vary the cutoff point from districts of twelve representatives and greater to districts of twenty representatives or greater (nine measures altogether). For each of these twelve specifications, we conducted the same process as we did with seat ratio: we ran the baseline model and a model akin to Model 2 (but with a different measure of variation) and computed the level of permissibility for each. We then produced shaded figures of the two effects akin to Figure 4 reported above, and finally produced a figure of the differences between predicted permissibility under the baseline model and that under the model that includes variation, per Figure 5 above. The results of the nine alternative thresholds are very similar, and thus, in the interest of space, we present the gaps of the smallest threshold (twelve seats) along with the three other measures, four alternative measures altogether. The raw results of the four models are reported in Table A2, Models 3-6. In all models the raw effect of variation is consistent with that of seat ratio and reaches standard levels of statistical significance.

Figure A1 in the on-line appendix presents this last step of the analysis for the four variation measures: the proportion of seats in the upper half of districts (panel a), the standard deviation of district magnitude (b), the proportion of seats elected in districts of twelve seats or greater (c), and skewness (d). In each panel on the horizontal axis is the relevant measure of variation and on the vertical axis is the difference in predicted permissibility. As in previous figures, the figure includes the thirty-eight cases of districted PR.

As can be seen in the figure, in all specifications the results are consistent with those found in our main analysis: the electoral system is less representative than the literature infers where there is little variation in magnitude (on the left) and more permissible than

inferred where districts vary and where a substantial part of legislature is elected via large districts.

Additionally, we repeated our analysis adding various alternative control variables (similar to those reported in Carey and Hix's (2011) Table 2), twelve altogether. As in Equation (2), all specifications (described below) include ENPV, magnitude of median district (logged), seat ratio, and interactions of the latter two with ENPV. In all models seat ratio had a similar effect as the one reported so far and reached standard levels of statistical significance, both in itself and interacted with ENPV. In particular, when we included a control for Single Transferable Vote, ethnolinguistic fractionalization (Fearon 2002), Freedom House score, formal threshold, bicameralism, parliamentary/presidential regime, post-colonial dummy (Hadenius and Teorell 2005) and region, as well as no additional controls (with the exception of the "regular" ones of SMD, MMM, fused vote and upper tier) results for both central tendency and variation remained almost entirely unchanged. When we included Polity IV score (9 or 10) or the country's size of population (United Nations Statistics Divisions), results for both central tendency and variation increased in their magnitude. When we controlled for federalism or percent of population living in urban communities the effect of the constitutive term of the median district lost statistical significance.

Lastly, we reran all models reported in Table A2 (the baseline model plus the five models that include variation) on the subset of the thirty-eight cases of districted PR only. The results (reported in Table A3) are consistent with those reported above. In all models indicators of variation reach standard levels of statistical significance. The median district, however, loses statistical significance in some of the models.

As we note above, we do not claim that one particular measure of variation in magnitude is the ultimate way to go. Rather, we call attention to the almost entirely overlooked within-country variation and particularly long upper tail of large districts and the effect it has on representation. It is therefore reassuring that results are stable across measures of variation as well as across model specifications.

6. Application

One might wonder to what extent variation in district magnitude affects additional political outcomes. As mentioned above, two past studies find an effect of various aspects of variation in district magnitude on the electoral outcomes. Monroe and Rose (2002) demonstrate that the variance relative to the median affects the electoral fortunes of rural parties, and Kedar et al. (2016) find that the lower tail of small districts affects representational inequality. In the analysis that follows, we apply the seat ratio -- the measure introduced in this study capturing the upper tail of large districts -- to analysis of the psychological effect of Duverger's theory: how social heterogeneity translates to the party system in the electorate.

We follow the footsteps of Clark and Golder's (2006) compelling study of Duverger's theory. In it, the authors call attention to the critical role of social forces in shaping party systems under Duverger's theory. We first conduct an analysis akin to Clark and Golder on our own data and find similar results. We then add seat ratio to the analysis. In particular, we estimate:

(4) $$ENPV_i = \beta_0 + \beta_1 ELF_i + \beta_2 ln(AvgD_i) + \beta_3 ELF_i ln(AvgD_i) + \beta_4 SR_i + \beta_5 ELF_i SR_i + \beta_6 UpperTier_i + \beta_7 ELF_i UpperTier_i + \beta_8 ENPres_i + \beta_9 Proximity_i +$$

$\beta_{10} ENPres_i Proximity_i + u_i$

that is identical to Clark and Golder's model with the exception of the two seat ratio variables. In it, *AvgD* is the average district magnitude (the central district measure used by the authors), *ELF* is ethno-linguistic fractionalization, *UpperTier* is the proportion of seats in parliament elected in upper tier, *ENPres* is the effective number of presidential candidates, and *Proximity* is the time proximity between legislative and presidential elections (which takes the value of zero under parliamentary regime).

Needless to say, their study includes additional analyses and important findings. We do not elaborate on it here simply because these parts are less relevant for the focus of our study. Our goal in this application is not to prove them wrong but rather to examine how the logic introduced in the current study might be incorporated into other studies such as theirs.

Figure 6 below presents the results of this analysis. On the vertical axis is the effect of ELF on ENPV and on the horizontal axis is average district magnitude (logged). The solid line describes the effect for cases with large variation (the 90th percentile) and the dashed line for those with small variation (10th percentile). The estimated effects are similar, yet their confidence intervals reveal a difference. For cases with large variation, the effect is statistically different from zero only above certain average magnitude. For cases with small variation, however, the effect is not statistically different from zero along the entire range. In other words, only when an upper tail of large districts is present does greater social heterogeneity translate to more parties in the electorate above a certain threshold of an average magnitude. We follow Clark and Golder's interpretation of this effect as the psychological effect. Note that the effect found here for cases with high variation only is found by Clark and Golder for the entire range. Although the dependent variable is different,

this study is consistent with our other findings: other things equal, long upper tail of large districts improves representation.

-- Figure 6 here --

7. Implications for Institutional Design and Measurement

Many democracies are characterized by an electoral system of varying district magnitude. Within the same country some districts elect few representatives while others elect many with a gap between the two ends of up to twenty-fold and greater, yet other states are characterized by districts of similar magnitude. This study is the first that examines the effect of this variation and particularly the presence of large districts on the permissibility of the electoral system and thus on representation. We argue and demonstrate that districts of identical magnitude produce permissibility inferior to that of a combination of small and large districts. Indeed, the working assumption that electoral systems are similar if they are similar on average is often a perilous one. Other things equal, a tail of large districts leads to higher level of permissibility and hence representativeness. Permissibility is one aspect of electoral systems, one that is particularly relevant in institutions designed for heterogeneous and multiple-cleavage societies. Minority representation, cabinet stability and governability, accountability, and disproportionality are additional dimensions by which students of comparative politics evaluate electoral outcomes. It is likely that different aspects of the distribution of districts affect different political and economic outcomes. Further unpacking the weighty explanatory power that political scientists attribute to district magnitude will likely enhance our understanding of electoral politics in general and representation in particular.

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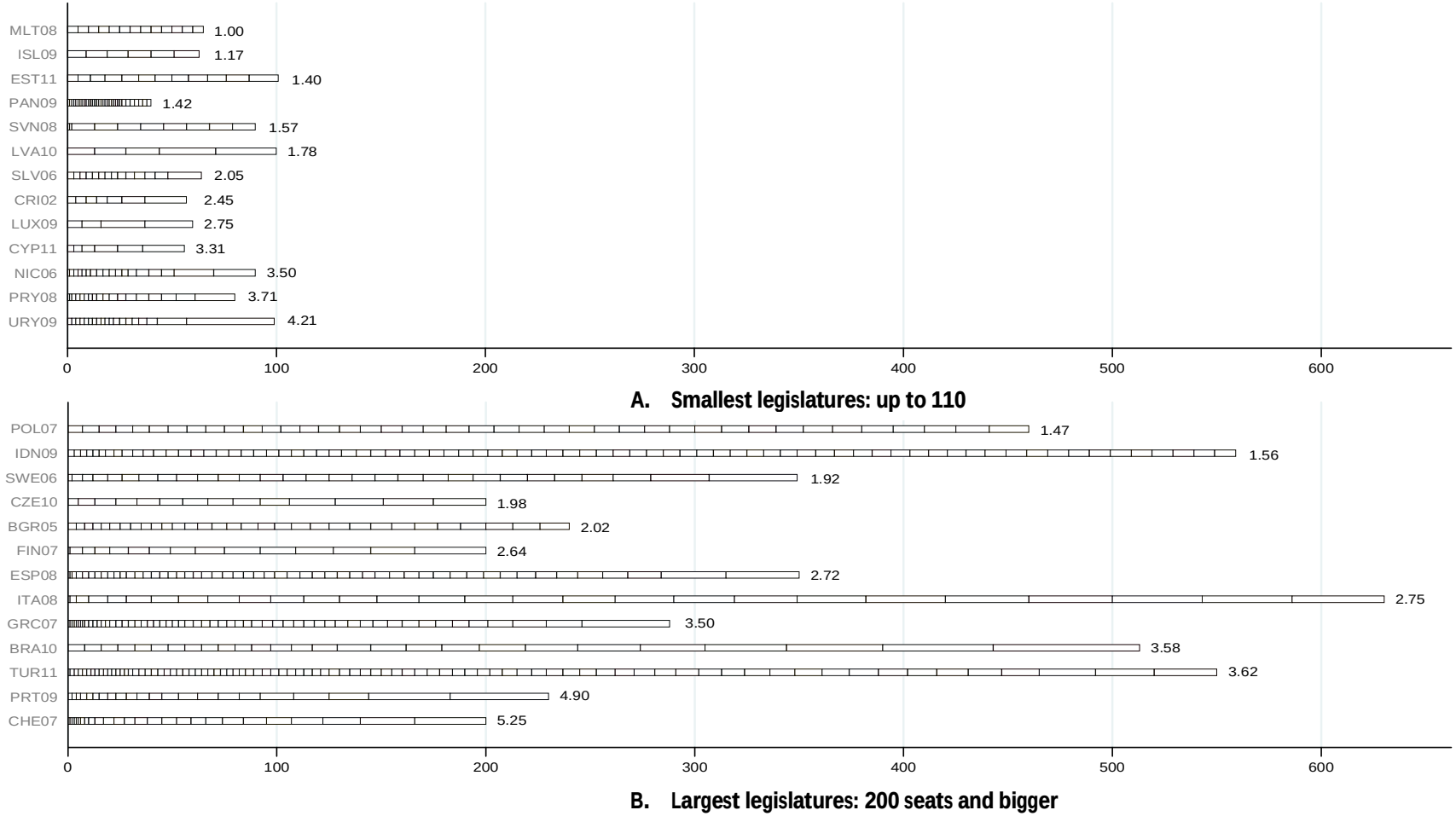
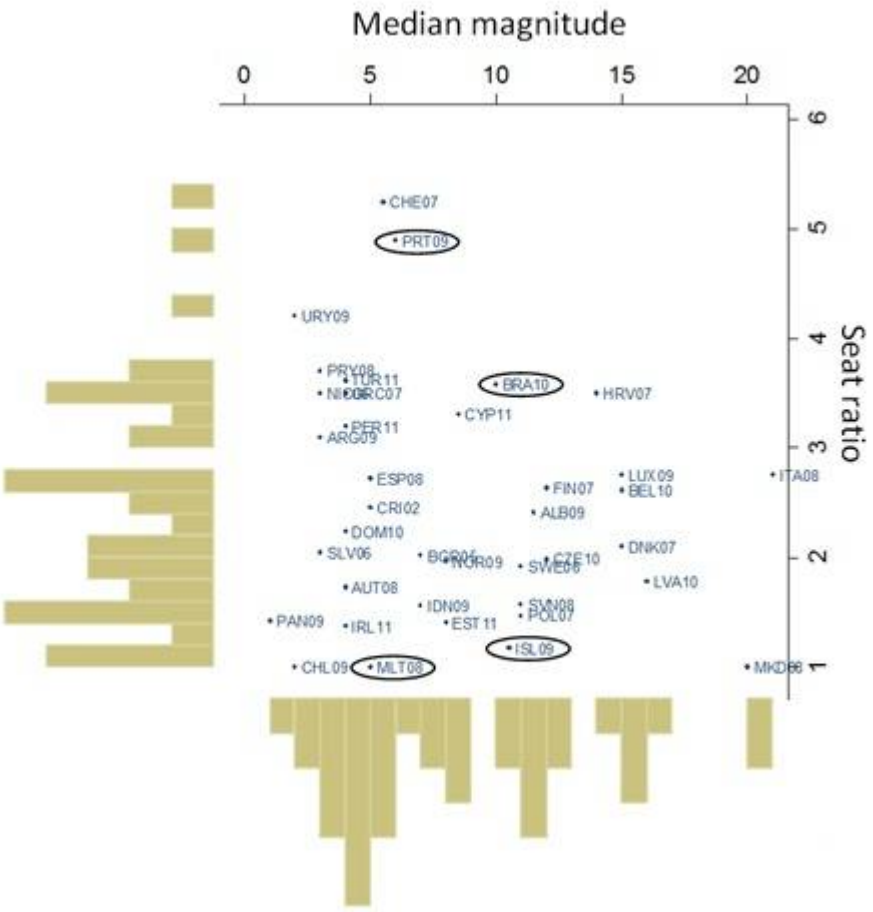
Figure 1. District magnitudes within countries

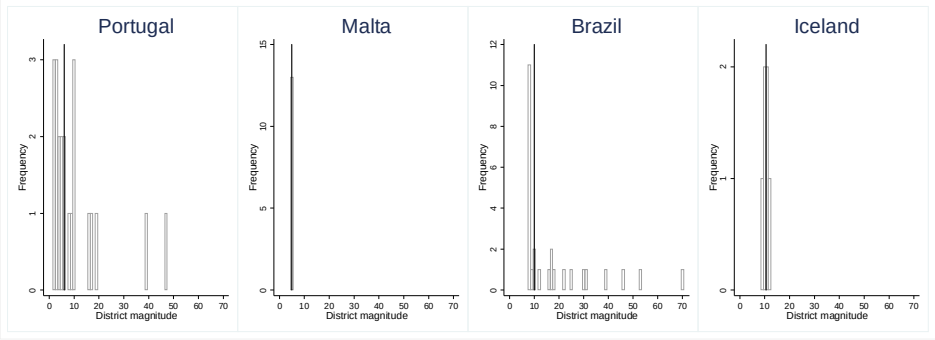


Note. The figure presents districts in parliament for all districted PR's, from smallest to largest, such that each rectangle is proportionate to the number of seats in the district. Cases are organized in ascending order by the share of seats elected in the top half of districts (values of seat ratio are noted to the right of the bars). For convenience, cases are divided to two groups by size of parliament.

Figure 2. Electoral systems: within country variation



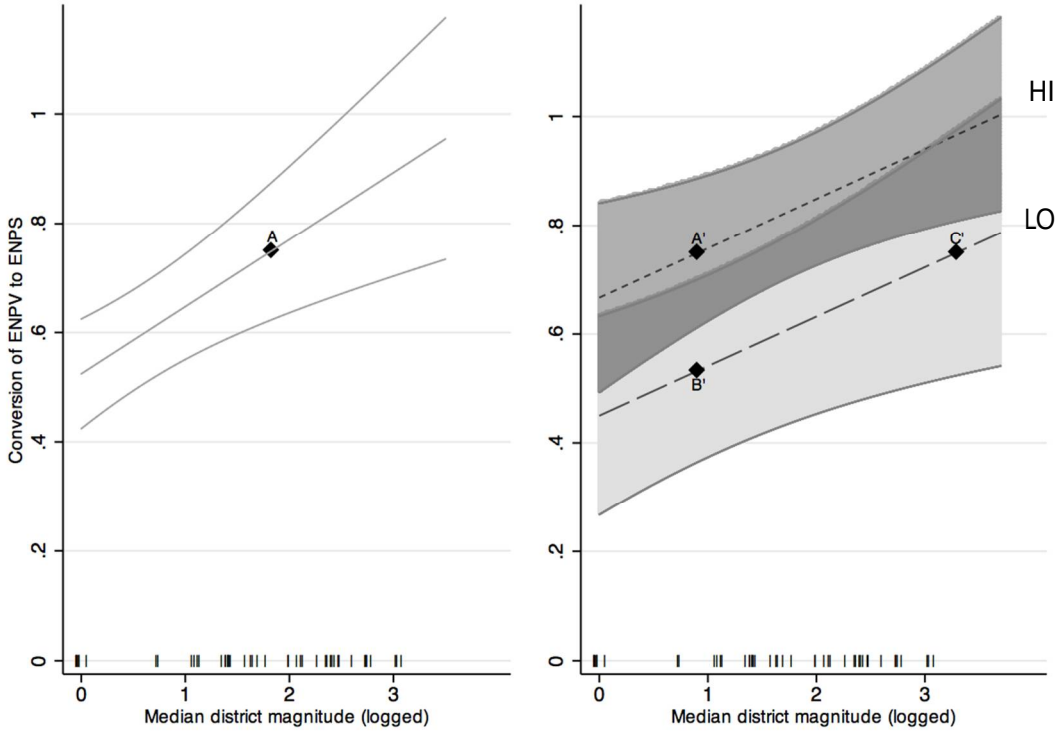
(a)



(b)

Note. Panel (a) presents a scatterplot of median district magnitude and seat ratio in districted PR systems. Histograms of the two variables are at the bottom and left margin. Panel (b) presents histograms of district magnitudes in four districted PR systems. The black bold line marks the median district magnitude.

Figure 3. Effective number of parties: a two dimensional picture



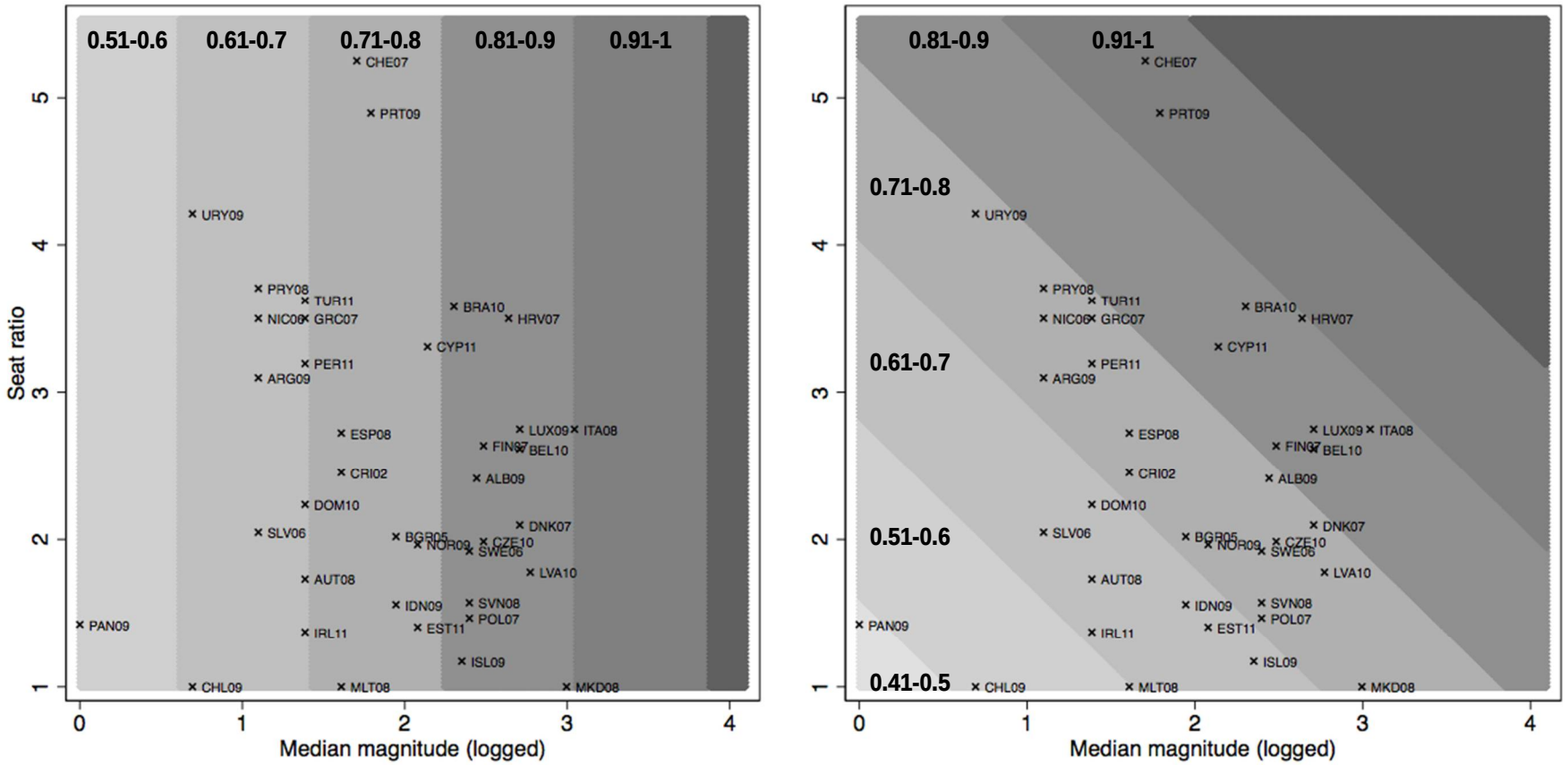
a. Baseline model
A=(6.27,0.75)*

b. Our model
A'=(2.49,0.75)
B'=(2.49,0.53)
C'=(27.01,0.75)

Note. Both panels present the conversion of ENPV to ENPS (on the vertical axis) as a function of (logged) median district magnitude (on the horizontal axis). Results in panel (a) are based on Equation 1a and those in panel (b) on Equation 2a. In panel (b) the upper (short) dashed line presents this effect for the 90th percentile of SR and the lower (long) dashed line – for the 10th percentile. Both lines are accompanied by 95% confidence intervals. The rug plot marks the (logged) median district magnitudes among districted PR and MMM cases.

*Horizontal coordinates are given in terms of unlogged district magnitude.

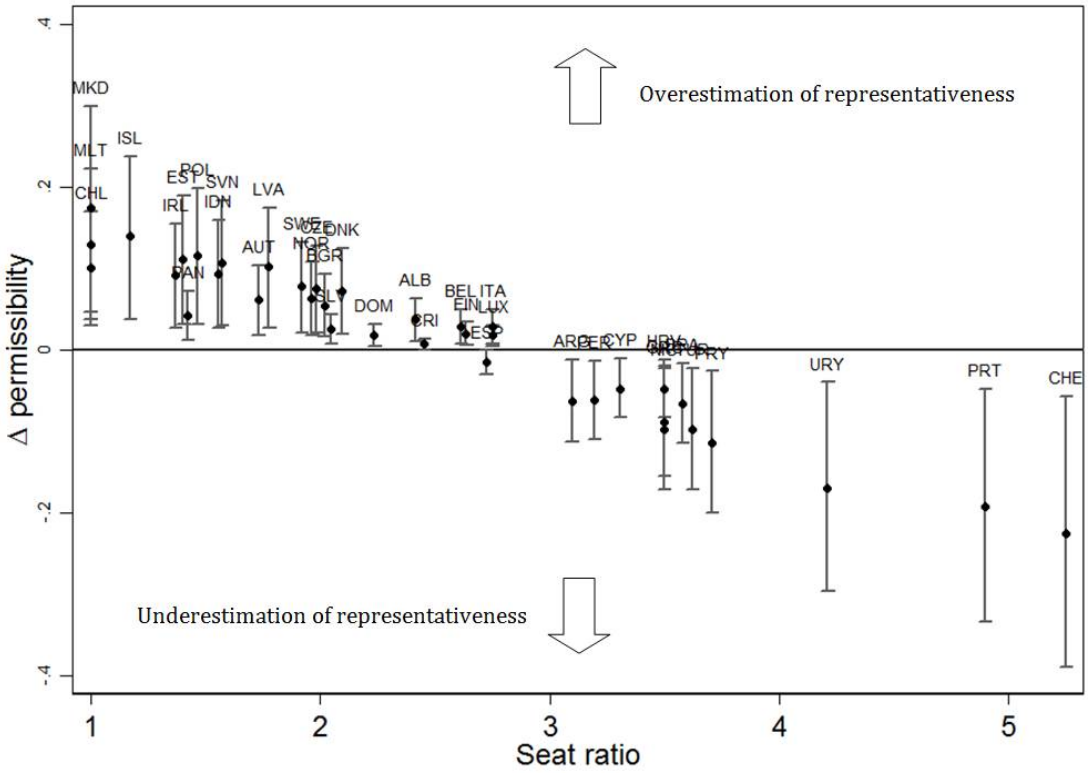
Figure 4. Effective number of parties: a three dimensional picture



a. Baseline model **b. Our model**

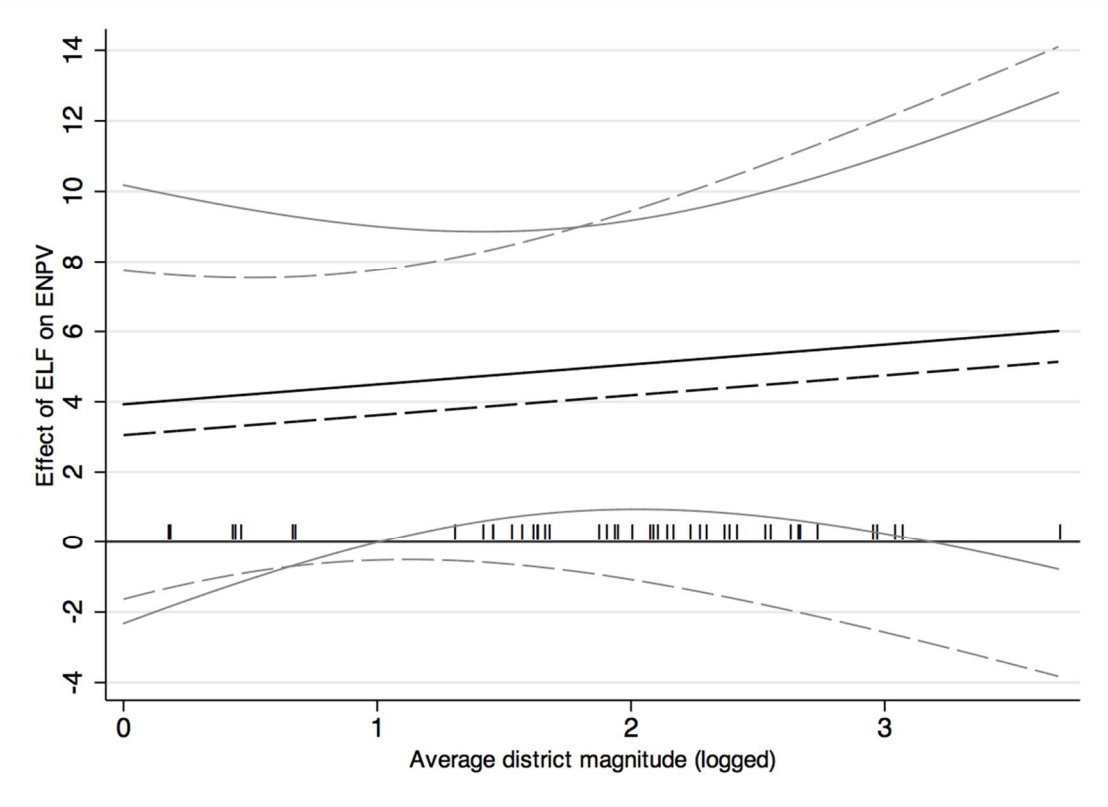
Note. Panels (a) and (b) present results based on Equations 1a and 2a, respectively. The gray areas are the estimated conversion values of ENPV to ENPS, collapsed into 0.1-width intervals. Darker shade denotes greater permissibility. The scatter of cases overlaid includes districted PR's, as in Figure 1.

Figure 5. Gaps in predicted permissibility: Comparison between baseline and our model



Note. On the vertical axis is the gap between predicted permissibility as calculated by Equation 1a (baseline model) and that calculated by Equation 2a. Positive differences imply that the baseline model overestimates conversion compared to our model. The whiskers denote 95% confidence intervals. The models themselves (estimated on all cases included in the study) are presented in Table A2 in the appendix.

Figure 6. Effective number of parties: Application of Clark and Golder



Note. The figure presets the results of a model akin to Clark and Golder's Equation 4 (2006, p. 695) plus seat ratio and its interaction with Fearon's ELF, run on our data (N=54). Solid line represents the 90th percentile seat ratio and dash line represents the 10th percentile of seat ratio.

On-line Appendix

Table A1. Countries and election years

Country	Election Year	Country	Election year
Albania	2009	Malta	2008
Argentina	2009	Mexico	2009
Australia	2010	New Zealand	1993
Austria	2008	Nicaragua	2006
Belgium	2010	Norway	2009
Botswana	2009	Panama	2009
Brazil	2010	Paraguay	2008
Bulgaria	2005	Peru	2011
Canada	2011	Poland	2007
Chile	2009	Portugal	2009
Costa Rica	2002	Slovenia	2008
Croatia	2007	South Africa	2009
Cyprus	2011	South Korea	2008
Czech Rep.	2010	Spain	2008
Denmark	2007	Sweden	2006
Dominican Republic	2010	Switzerland	2007
El Salvador	2006	Taiwan	2008
Estonia	2011	Turkey	2011
Finland	2007	UK	2010
France	2007	Uruguay	2009
Ghana	2008	USA	2008
Greece	2007		
Guatemala	2007		
Iceland	2009		
India	2009		
Indonesia	2009		
Ireland	2011		
Italy	2008		
Japan	2009		
Latvia	2010		
Lithuania	2012		
Luxemburg	2009		
Macedonia	2008		

Note. Table presents the 54 countries and election years for which the data of this research retrieved.

Table A2. Institutional mediation of social heterogeneity (MedD)

	(1) Baseline model	(2) Including SR	(3) Including % top quartile	(4) Including SD(D)	(5) Including skewness	(6) Including %>12
ENPV	0.524***	0.368***	0.346*	0.480***	0.022	0.504***
	(0.051)	(0.129)	(0.196)	(0.066)	(0.245)	(0.060)
MedD(Logged)	-0.430***	-0.289*	-0.427**	-0.269*	-1.167***	-0.167
	(0.154)	(0.155)	(0.160)	(0.159)	(0.410)	(0.184)
ENPV X MedD	0.123***	0.091**	0.126***	0.074**	0.316***	0.055
	(0.035)	(0.035)	(0.040)	(0.035)	(0.100)	(0.041)
Seat Ratio		-0.312*				
		(0.178)				
ENPV X Seat Ratio		0.082*				
		(0.046)				
% Top quartile			-1.617			
			(1.229)			
ENPV X top quartile			0.415			
			(0.346)			
SD(D)				-0.033**		
				(0.015)		
ENPV X SD(D)				0.016***		
				(0.005)		
Skewness					-0.259*	
					(0.141)	
ENPV X Skewness					0.068**	
					(0.032)	
%>12						-1.164*
						(0.691)
ENPV X %>12						0.310**
						(0.147)
SMD	-0.466***	-0.446**	-0.460*	-0.415**		-0.489***
	(0.169)	(0.215)	(0.230)	(0.190)		(0.174)
MMM	-0.906***	-1.012***	-0.949***	-0.937***	-0.861***	-0.883***
	(0.214)	(0.309)	(0.268)	(0.218)	(0.197)	(0.204)
Fused Ballot	0.317	0.594	0.514	0.413*	0.649	0.560*
	(0.274)	(0.357)	(0.324)	(0.222)	(0.405)	(0.308)
% Upper Tier	1.781***	1.701**	1.601**	1.352**	1.329**	1.238**
	(0.495)	(0.652)	(0.598)	(0.619)	(0.507)	(0.612)
Constant	1.076***	1.647***	1.756**	1.190***	3.021***	1.175***
	(0.193)	(0.487)	(0.686)	(0.251)	(1.038)	(0.245)
N	54	54	54	54	42	54
R-squared	0.940	0.949	0.944	0.956	0.948	0.951

Note. Dependent variable: ENPS. Model (1) is the baseline model per Equation 1. It is used for generation of Figures 3a and 4a. Models (2-6) include alternative measures of variation. Model 2 follows Equation 2 and is used for generation of Figures 3b and 4b. Models 3 to 6 are used for the generation of Figure A1. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table A3. Institutional mediation of social heterogeneity (MedD) – Districted PR only

	(1) Baseline model	(2) Including SR	(3) Including % top quartile	(4) Including SD(D)	(5) Including skewness	(6) Including %>12
ENPV	0.484**	0.189	-0.093	0.501***	0.005	0.754***
	(0.205)	(0.172)	(0.298)	(0.144)	(0.235)	(0.173)
MedD(Logged)	-0.521*	-0.353	-0.748**	-0.137	-1.294***	0.394
	(0.307)	(0.268)	(0.325)	(0.255)	(0.433)	(0.448)
ENPV X MedD	0.145*	0.106*	0.212**	0.046	0.314***	-0.093
	(0.082)	(0.062)	(0.081)	(0.065)	(0.093)	(0.101)
Seat Ratio		-0.552***				
		(0.163)				
ENPV X Seat Ratio		0.142***				
		(0.036)				
% Top quartile			-4.034***			
			(1.345)			
ENPV X top quartile			1.050***			
			(0.319)			
SD(D)				-0.079***		
				(0.026)		
ENPV X SD(D)				0.023***		
				(0.005)		
Skewness					-0.494**	
					(0.180)	
ENPV X Skewness					0.112***	
					(0.035)	
%>12						-1.911**
						(0.791)
ENPV X %>12						0.491***
						(0.149)
% Upper Tier	2.318***	2.773***	2.509***	2.455***	2.034***	2.591***
	(0.590)	(0.769)	(0.675)	(0.665)	(0.640)	(0.617)
Constant	1.218	2.323***	3.378***	1.121*	3.404***	0.213
	(0.746)	(0.734)	(1.189)	(0.562)	(1.171)	(0.715)
Observations	38	38	38	38	35	38
R-squared	0.932	0.959	0.953	0.959	0.950	0.955

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

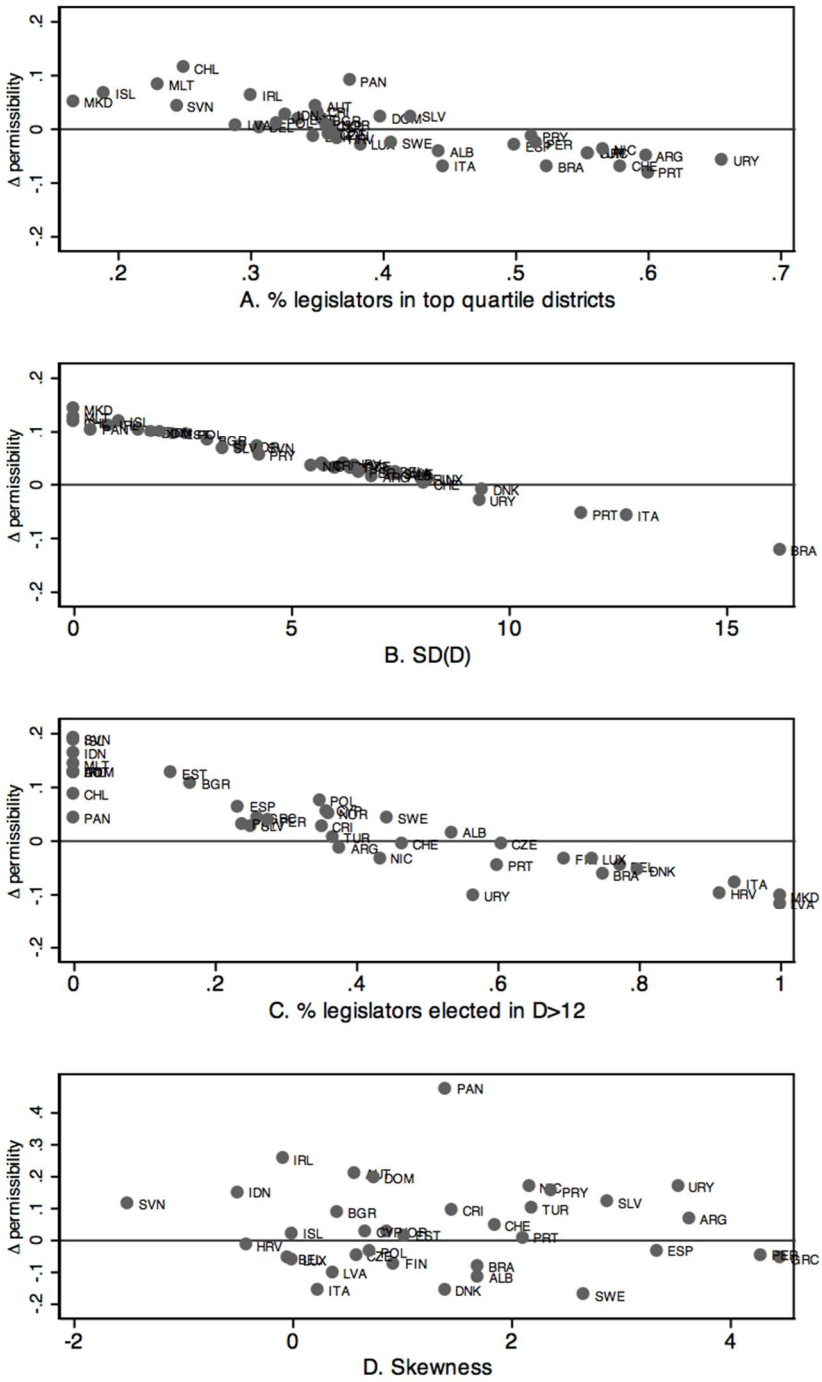
Table A4. Institutional mediation of social heterogeneity (MedD) with triple interaction

	(1) Triple interaction
ENPV	0.675***
	(0.080)
MedD(Logged)	0.373
	(0.290)
Seat ratio	0.128
	(0.222)
ENPV X MedD	-0.128*
	(0.068)
ENPV X Seat Ratio	-0.074*
	(0.043)
MedD X Seat Ratio	-0.282**
	(0.121)
ENPV X MedD X Seat Ratio	0.098***
	(0.028)
SMD	-0.570**
	(0.271)
MMM	-0.979***
	(0.283)
Fused Ballot	0.358
	(0.377)
% Upper Tier	2.493***
	(0.679)
Constant	0.762
	(0.469)
Observations	54
R-squared	0.960

Note. Dependent variable: ENPS. Model 1 is used for generation of Figures A2b and A3b. Robust standard errors in parentheses.

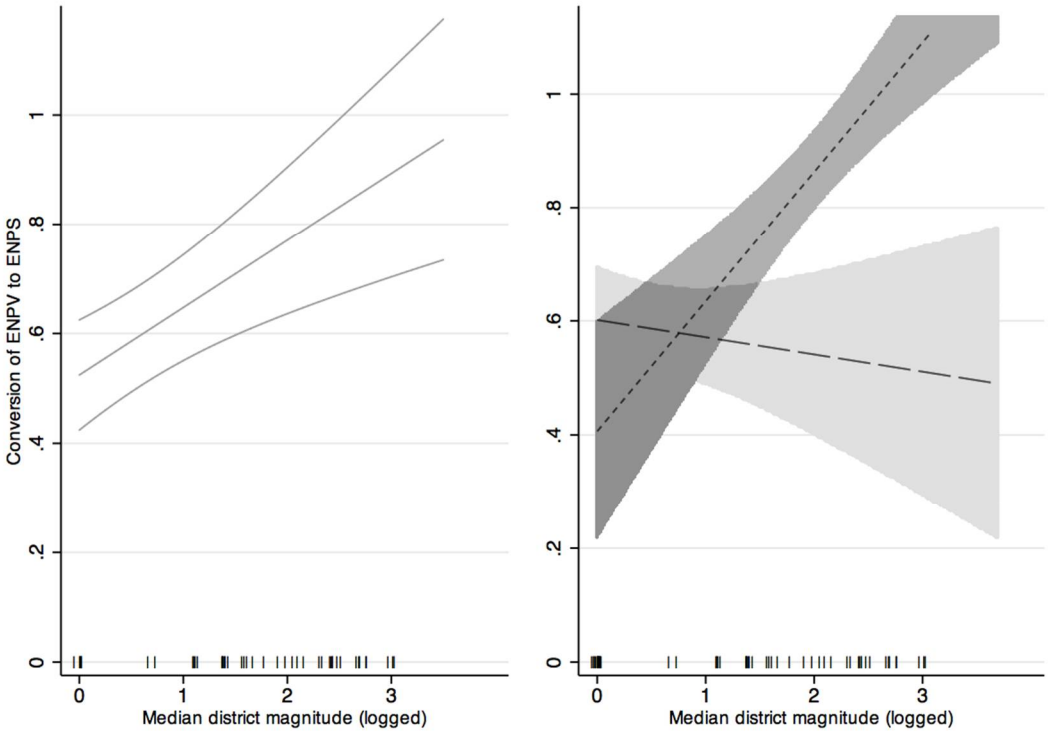
*** p<0.01, ** p<0.05, * p<0.1

Figure A1. Gaps in predicted permissibility: alternative measures of variation



Note. On the vertical axis is the difference between predicted permissibility according to the baseline model (Equation 1a) and the model incorporating variation while each panel uses an alternative measure of variation. These are the proportion of legislators elected via largest 25% of districts (panel a), standard deviation of district magnitude (panel b), proportion of legislators elected in districts greater than twelve seats (c), and skewness of district magnitude (panel d). Positive difference implies that the baseline model overestimates permissibility compared to the model that incorporates variation. The models themselves (estimated on all cases included in the study) are presented in Table A2.

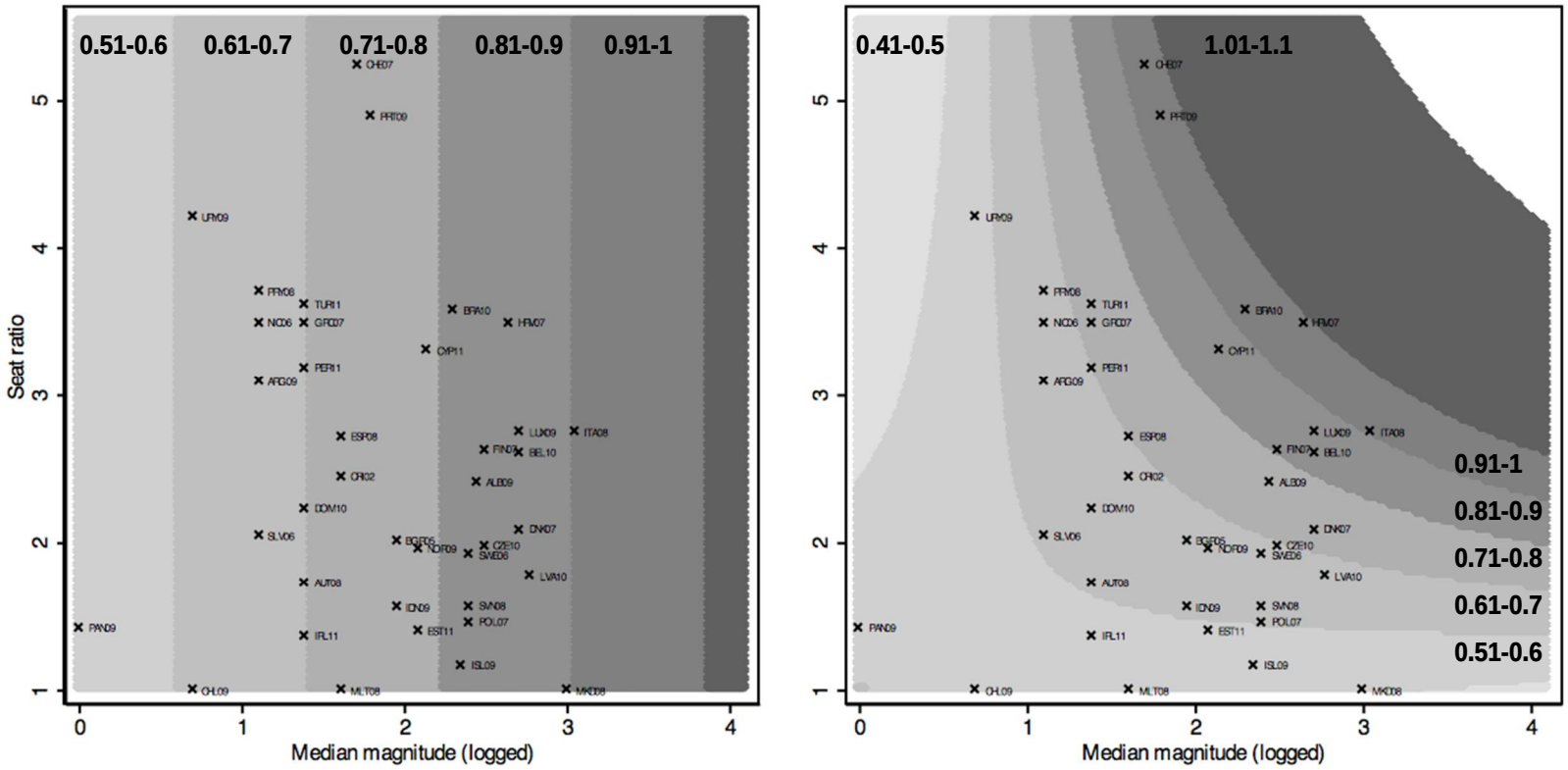
Figure A2. Effective number of parties: a two dimensional picture (triple interaction)



a. Baseline model **b. Our model**

Note. Both panels present the conversion of ENPV to ENPS (on the vertical axis) as a function of (logged) median district magnitude (on the horizontal axis). Results in panel (a) are based on Equation 1a and those in panel (b) on the marginal effect of a triple-interaction model. In panel (b) the upper (short) dashed line presents this effect for the 90th percentile of SR and the lower (long) dashed line – for the 10th percentile. Both lines are accompanied by 95% confidence intervals. The rug plot marks the (logged) median district magnitudes among districted PR and MMM cases.

Figure A3. Effective number of parties: a three dimensional picture (triple interaction)



a. Baseline model **b. Our model**

Note. Panels (a) and (b) present results based on Equations 1a and on the marginal effect of a triple-interaction model, respectively. The gray areas are the estimated conversion values of ENPV to ENPS, collapsed into 0.1-width intervals. Darker shade denotes greater permissibility. The scatter of cases overlaid includes districted PR's, as in Figure 1.